別紙第１号様式（乙） No. 1

博　士　論　文　の　要　旨

氏　名（本籍）　Amjid Ali （パキスタン）　　印

博士論文題名

ϕ -Haar Wavelet Methods for Numerical Solutions of Fractional Differential Equations

（ϕ-ハールウェーブレットによる分数次微分方程式の数値解法）

要旨

Fractional differential equations are the best way to model many real-world physical phenomena. Apart from modeling solution strategies and their repercussions are essential for determining critical points where a significant divergence or bifurcation begins. Therefore, high-precision solutions are always required. There are many different definitions of fractional derivatives in literature, one simple way to cope with this variation is to combine those concepts by considering fractional derivatives of a function with respect to another function ψ. Fractional differential equations with ψ-Caputo derivative provide more flexible models, in the sense that by a proper choice of function ψ, hidden features of the real-world phenomena could be extracted. The main objective of this research is to develop reliable and proficient numerical methods for solving linear and nonlinear fractional differential equations involving ψ-Caputo fractional derivative. In this work w,e introduced a new numerical method, the ψ-Haar wavelets operational matrix method, for solving fractional differential equations. We derived an operational matrix for the numerical approximation of ψ-fractional differential equations. We extended the method to nonlinear ψ-fractional differential equations by using the quasilinearization technique. The quasilinearization techniques convert the fractional nonlinear differential equation to a fractional discretized differential equation. The method is a simple and good mathematical tool for finding solutions of nonlinear ψ-fractional differential equations. The operational matrix approach offers less computational complexity. The error analysis of the proposed method is carried out. The accuracy and efficiency of the method are verified through numerical examples.

In Chapter 1 we have provided a brief introduction to fractional calculus.

In the first sections of Chapter 2, we introduced some special functions that were required for the development of our results. In the second section, the quasilinearization technique is discussed which is used for linearizing nonlinear problems. In the third and fourth sections, some fundamental concepts, and definitions from fractional calculus and ψ- fractional calculus are provided. In section five, Haar wavelets and function approximation by Haar wavelets are given. Also, we constructed an operational matrix, called the ψ-Haar wavelet operational matrix. In the last section of chapter 2, the error analysis of the numerical scheme based on the ψ-Haar wavelets operational matrix is discussed in-depth.

In chapter 3, we established a numerical scheme based on ψ-Haar wavelet operational matrices for solving linear and nonlinear initial value ψ-fractional differential equations. The quasilinearization technique is applied to convert the fractional nonlinear differential equation to a fractional to discretized differential equation. ψ-Haar wavelet method is applied at each iteration of the quasilinearization technique to get the approximate solution. We give some numerical examples utilizing the ψ-Haar wavelet operational matrix method to approximate the numerical solutions of linear and non-linear initial value fractional differential equations.

博　士　論　文　の　要　旨

氏　名　Amjid　Ali　　印

The wavelet-based method reduces the problem to a system of algebraic equations. The numerical results obtained are compared with exact solutions by tabulating their absolute error and by comparing their respective graphs. It is worth mentioning that results obtained, agree well with exact solutions even for small number of collocation points. More accurate results are obtained by increasing the level of resolution J.

ψ-Haar wavelet technique has been extended, in Chapter 4, for linear and nonlinear fractional boundary value problems. The procedure of implementation of the method for general fractional differential equation has been introduced. This approach relies on the ψ-Haar wavelet operational integration matrices. The ψ-operational matrices are used to convert the ψ-Fractional Differential Equations to an algebraic system of equations.

To handle the nonlinear case, we use the quasilinearization technique to transform the nonlinear ψ-fractional differential equation into linearized form and then ψ-Haar wavelets technique is applied in succession. The results obtained by the numerical scheme based on the ψ-Haar wavelet operational matrix method are compared with exact solutions. It has been observed that the results are in good agreement with the exact solution. The proposed method is a good and simple mathematical technique for numerically solving non-linear ψ-fractional differential equations. The operational matrix method is computationally more efficient. Several linear and non-linear boundary value problems are discussed to demonstrate the applicability, efficiency, and simplicity of the method. The method is convenient for solving linear and nonlinear initial value problems as well as boundary value problems. It has been observed that the method gives more accurate results while increasing the level of resolution.

In Chapter 5, a numerical method based on the two-dimensional ψ-Haar wavelets is discussed for numerical solutions of arbitrary order ψ-fractional partial differential equation. The method is applied to fractional initial and boundary value problems with constant coefficients and variable coefficients. We considered the time-fractional telegraph equation, linear fractional diffusion equation, and convection-diffusion equation with ψ-Caputo fractional derivative as test problems. A comparison of the approximate and exact solutions is carried out, results are given in the graphical and tabular form. It is observed that the solution becomes more accurate by increasing the level of resolution of the method.

Operational matrices approach has been applied for the first time to solve ψ-fractional partial differential equations. ψ-Haar wavelets quasilinearization technique can be extended for ψ-fractional nonlinear partial differential equations. The proposed method may be extended to other wavelet bases, including Legendre wavelets, Chebyshev wavelets, CAS wavelets, Sine Cosine wavelets, and Gegenbauer wavelets. Also, other ψ-fractional operators can be introduced and utilized.