

## COMPARATIVE STUDY OF LINEAR AND NON-LINEAR THEORIES ONE-DIMENSIONAL CONSOLIDATION OF THICK CLAY LAYERS

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**ABSTRACT:** The classical theory of consolidation developed by Terzaghi is based on linear void ratio-effective stress relationship, thin layer of clay with negligible self weight, infinitesimal strain, constant volume (1+void ratio) and constant coefficients of permeability, volume compressibility and consolidation. This paper presents a simplified theory of non-linear one-dimensional consolidation of a thick clay deposit considering linear void ratio-log effective stress relationship, self weight of soil, constant volume (1+void ratio), thickness of clay layer and coefficient of consolidation but neglecting the slight variation of initial void ratio with depth. The proposed equation for consolidation of the deposit is solved numerically by the finite difference method and the results compared with those of the conventional linear theory. The results indicate that the variation of degree of settlement with time is relatively large while the variation of the degree of dissipation of excess pore pressure with time is relatively small in the case of thick layer of clay compared to those for thin layer. The variations of degrees of settlement and the dissipation of pore pressures are sensitive to the magnitude of applied load relative to the thickness of the deposit unlike in the conventional theory for thin layer. The isochrones in the case of pervious top and pervious bottom boundary conditions are slightly skewed in contrast to symmetrical isochrones of conventional linear theory.

**Keywords:** Consolidation, non-linear theory, thick layer; constant coefficient of consolidation, stress ratio, dissipation of pore pressure.

### 1. INTRODUCTION

The conventional one-dimensional consolidation theory developed for thin layers of fine grained soils by Terzaghi neglects the effect of self weight of the soil and assumes linear relationship of void ratio and effective stress, infinitesimal strain and constant coefficients of permeability, volume compressibility and hence consolidation. However, the void ratio is not proportional to effective stress and the coefficients of compressibility and permeability decrease during consolidation for a relatively large applied stress increment. More over, in a thick clay deposit the self weight of clay and the corresponding variation of initial in situ effective stress with depth are significant.

Richart (1957) reviewed the theory of consolidation and concluded that the effect of considering void ratio as a variable did not significantly change the consolidation-time characteristics of consolidation by vertical flow. The non-linear theory of one-dimensional consolidation

developed by Davis and Raymond (1965) considering linear void ratio-log effective stress relationship, is valid only for a thin layer of clay. Gibson et al. (1981) presented a finite strain non-linear one-dimensional consolidation theory for thick homogeneous clays by considering the self weight of soil, variation of void ratio with depth and Lagrangian and convective coordinate system. The Lagrangian coordinate system is cumbersome to use. Vaid (1985) presented a solution for vertical non-linear consolidation neglecting self weight, under constant rate of loading and introduced a new dimensionless parameter to quantify the magnitude of difference between linear and non-linear results. Analytical solution developed by Lee et al. (1992) for the problem of one-dimensional consolidation of layered soils found that the effects of coefficients of permeability and compressibility cannot be embodied into the coefficient of consolidation of a soil layer in a multi-layered system unlike in a single layer system and that the stiffness of soil layer plays an important role on the

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consolidation of layered system. Solution proposed by Xie et al. (2002) for non-linear one-dimensional consolidation of two-layered soil is based on effective stress being constant with depth. Lekha et al. (2003) present solution for vertical consolidation of a layer of finite thickness, considering the variations in compressibility and permeability of soil but neglecting the effect of self weight of the soil. Xie and Leo (2004) derived large strain solutions for one-dimensional consolidation of both thin and thick clay layers. Chen et al. (2005) developed the one-dimensional nonlinear consolidation theory for multi-layered soil by differential quadrature method. Zhuang et al. (2005) presented a semi-analytical solution for one dimensional consolidation of clays with variable compressibility and permeability and found that the ratio  $C_c/C_k$  (the slopes of  $e - \log \sigma'_v$  and  $e - \log k$  respectively, where  $e$  is the void ratio,  $k$  the coefficient of permeability,  $\sigma'_v$  the effective stress) determine the need to consider the effect of nonlinearity. The linear and non-linear theories of consolidation proposed by Conte and Troncone (2006 and 2007) are applicable for thin layers subjected to general time dependent loading. Abbasi et al. (2007) developed a finite difference approach for consolidation with variable compressibility, permeability and coefficient of consolidation. A spectral method is presented by Walker et al. (2009) for analysis of vertical and radial consolidation in multilayered soil with PVDs assuming constant soil properties within each layer. Xie et al. (2010) present a solution for one-dimensional consolidation of clayey soil with a threshold gradient.

2. ASSUMPTIONS, PROBLEM STATEMENT AND FORMULATION

A simple approximate theory of non-linear consolidation is proposed herein for a thick clay layer considering linear void ratio-log effective stress relationship but assuming coefficient of consolidation to be constant (the decrease in coefficient of permeability is proportional to the decrease in coefficient of volume change,  $m_v$ ), constant thickness of clay layer and constant initial void ratio along the depth and constant volume  $(1+e)$  during consolidation but accounting for the variation of initial effective stress with depth. Typical distribution of initial in-situ void ratio can be seen to be nearly constant with depth (Fig.1) for the subsoil at Changi Airport (Choa 1995). The proposed theory for a thick clay layer is an extension of the non-linear theory of consolidation developed by Davis and Raymond (1965) for a thin layer.

Considering linear  $e - \log \sigma'$  relationship, assuming  $(1+e)$  and coefficient of consolidation,  $c_v$ , to be constant during consolidation, the following general equation for one-dimensional consolidation is derived by Davis and Raymond (1965) as

$$\frac{1}{\sigma'_v} \frac{\partial \sigma'_v}{\partial t} = -c_v \left[ \frac{1}{\sigma'_v} \frac{\partial^2 u}{\partial z^2} - \left( \frac{1}{\sigma'_v} \right)^2 \frac{\partial u}{\partial z} \frac{\partial \sigma'_v}{\partial z} \right] \quad (1)$$

Assuming vertical stress ( $\sigma'_v$ ) to be constant with depth (equivalent to a thin layer), the above equation is further simplified by Davis and Raymond (1965) as

$$\frac{\partial w}{\partial t} = c_v \left[ \frac{\partial^2 w}{\partial z^2} \right] \quad \text{with } w = \log_{10} \left( \frac{\sigma'_v}{\sigma'_f} \right). \quad (2)$$

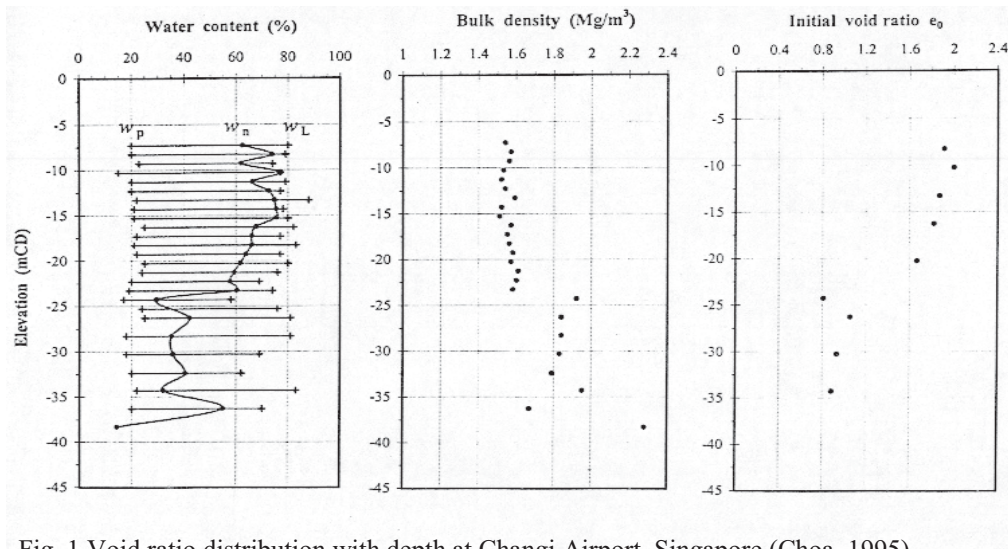


Fig. 1 Void ratio distribution with depth at Changi Airport, Singapore (Choa, 1995)

where  $\sigma'_v$  is the effective vertical stress at any time,  $t$  during consolidation,  $\sigma'_f$  the final effective vertical stress at the end of consolidation,  $u$  the excess pore water pressure and  $z$  the depth from surface. However, the initial effective stress,  $\sigma'_{vi}$  varies considerably with depth for a thick clay deposit. This variation is accounted for in the proposed theory as follows.

A homogeneous saturated clay layer of thickness,  $H$ , which is initially fully consolidated under its self weight, (Fig. 2), consolidates under a uniformly distributed load,  $q$ , applied on the top. At any depth,  $z$ , the effective vertical stress,  $\sigma'_v = (\gamma'z + q - u)$ , where,  $\gamma'$  is the submerged unit weight of the soil. Differentiating  $\sigma'_v$  with depth,  $z$ , twice one gets

$$\frac{\partial \sigma'_v}{\partial z} = \gamma' - \frac{\partial u}{\partial z} \quad (3)$$

$$\frac{\partial^2 \sigma'_v}{\partial z^2} = -\frac{\partial^2 u}{\partial z^2} \quad (4)$$

$$\text{Let } \omega = \log_{10} \left( \frac{\sigma'_v}{\gamma'.H} \right) \quad (5)$$

In Eq. (2), the final effective stress,  $\sigma'_f$ , is assumed to be constant with depth. However, in the proposed theory,  $\sigma'_f (=q + \gamma'.z)$  is not constant but increases with depth. Hence a new term  $\omega$ , instead of  $w$ , independent of depth dependent parameter is proposed herein.

Differentiation Eq. (5) with respect to time,  $t$ , and depth,  $z$ , one gets

$$\frac{\partial \omega}{\partial t} = \frac{0.434}{\sigma'_v} \cdot \frac{\partial \sigma'_v}{\partial t} \quad (6)$$

$$\frac{\partial \omega}{\partial z} = \frac{0.434}{\sigma'_v} \cdot \frac{\partial \sigma'_v}{\partial z} \quad (7)$$

$$\frac{\partial^2 \omega}{\partial z^2} = 0.434 \left[ -\frac{1}{(\sigma'_v)^2} \left( \frac{\partial \sigma'_v}{\partial z} \right)^2 + \frac{1}{\sigma'_v} \frac{\partial^2 \sigma'_v}{\partial z^2} \right] \quad (8)$$

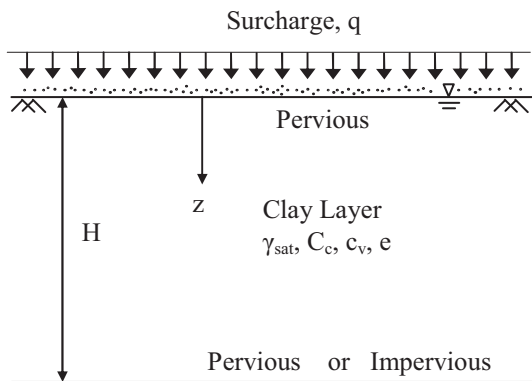


Fig.2 Problem definition sketch

Substituting the above equations into Eqn. (1), the following modified equation can be obtained in the proposed theory of non-linear consolidation of a thick clay layer,

$$\frac{\partial \omega}{\partial t} = c_v \left[ \frac{\partial^2 \omega}{\partial z^2} + \frac{\gamma'}{\sigma'_v} \frac{\partial \omega}{\partial z} \right] \quad (9)$$

$$\text{with } \omega = \log_{10} \left( \frac{\sigma'_v}{\gamma'.H} \right)$$

For non-linear thin layer (Davis and Raymond, 1965),

$$\frac{\partial w}{\partial t} = c_v \left[ \frac{\partial^2 w}{\partial z^2} \right] \quad \text{with } w = \log_{10} \left( \frac{\sigma'_v}{\sigma'_f} \right).$$

For a thin layer, the term with unit weight,  $\gamma'$ , vanishes and the equation (9) is of the same form as that of Davis and Raymond (1965) but with 'w' defined differently from 'omega'. Equation (9) is rewritten in non-dimensional form as

$$\frac{\partial \omega}{\partial T} = \left[ \frac{\partial^2 \omega}{\partial Z^2} + \frac{1}{10^{\omega}} \cdot \frac{\partial \omega}{\partial Z} \right] \quad (10)$$

where  $Z$  and  $T$  are the normalized parameters;  $Z = z/H$  and  $T = t/t_R$ ; reference time,  $t_R = H^2/c_v$ . The main differences among the three theories, viz. (i) conventional theory by Terzaghi (ii) Davis and Raymond (1965) and (iii) the proposed theory are summarized in Table 1.

### 3. INITIAL AND BOUNDARY CONDITIONS

Initial condition :  $t=0$ ;  $0 \leq z \leq H$ ;  $u(z,0) = q$ ;

$$\alpha(z,0) = \log_{10} \left( \frac{\gamma'.z}{\gamma'.H} \right) \quad \text{or } T=0 ; 0 \leq Z \leq 1 ;$$

$$\alpha(Z,0) = \log_{10}(Z) \quad (11)$$

Two boundary conditions, viz., (i) pervious top pervious base (PTPB) and (ii) pervious top impervious base (PTIB) need to be considered in the present case as the initial and final total and effective stresses vary with depth unlike in the case of consolidation of thin layers.

Boundary conditions:

$$\text{Pervious top, for } t>0; z=0; u(0, t) = 0; \alpha(0, t) = \log_{10} \left( \frac{q}{\gamma'H} \right)$$

$$\text{or for } T>0; Z=0; \omega(0, T) = \log_{10}(q^*) \quad (12)$$

$$\text{where } q^* = \frac{q}{\gamma'.H} \quad (13)$$

Table 1: Comparison of the proposed theory with the other theories

S. No	Terzaghi	Davis and Raymond (1965)	Proposed Theory
1	Linear e - $\sigma'_v$ relationship	Linear e - log $\sigma'_v$ relationship	Linear e - log $\sigma'_v$ relationship
2	$\sigma'_v$ is constant with depth	$\sigma'_v$ is constant with depth	$\sigma'_v$ varies with depth
3	k, $m_v$ and $c_v$ are constant	$c_v$ is constant; change in k proportional to change in $m_v$	$c_v$ is constant; change in k proportional to change in $m_v$
4	Thin layer	Thin layer	Thick layer
5	Self weight neglected	Self weight neglected	Self weight considered
6	The equation of consolidation is	The equation is	The equation is
	$\frac{\partial u}{\partial t} = c_v \left[ \frac{\partial^2 u}{\partial z^2} \right]$	$\frac{\partial w}{\partial t} = c_v \left[ \frac{\partial^2 w}{\partial z^2} \right]$ with $w = \log_{10} \left( \frac{\sigma'_v}{\sigma'_f} \right)$	$\frac{\partial \omega}{\partial t} = c_v \left[ \frac{\partial^2 \omega}{\partial z^2} + \frac{\gamma'}{\sigma'_v} \frac{\partial \omega}{\partial z} \right]$ with $\omega = \log_{10} \left( \frac{\sigma'_v}{\gamma' H} \right)$

The term,  $q^*$  combines the effects of two important terms, the magnitude of applied stress,  $q$  and the thickness of the layer,  $H$ .

Pervious base, for  $t > 0$ ;  $z = H$ ;  $u(H, t) = 0$  or

$$\omega(H, t) = \log_{10} \left( \frac{\gamma' H + q}{\gamma' H} \right)$$

or  $\omega(1, T) = \log_{10} (1 + q^*)$  (14)

Impervious base, for  $t > 0$ ;  $z = H$ ,

$$\frac{\partial u}{\partial z}(H, t) = 0 \text{ then } \frac{\partial \omega}{\partial z}(1, T) = \frac{0.434}{10^\omega}$$
 (15)

For  $t \rightarrow \infty$ ;  $0 \leq z \leq H$ ;

$$u(z, \infty) = 0; \omega(z, \infty) = \log_{10} \left( \frac{\gamma' z + q}{\gamma' H} \right) \text{ or}$$

$$\omega(Z, \infty) = \log_{10} (Z + q^*)$$
 (16)

Average degree of settlement,  $U_s$ , is

$$U_s = \frac{\int_0^H \log_{10} \left( \frac{\sigma'_v}{\sigma'_o} \right) dz}{\int_0^H \log_{10} \left( \frac{\sigma'_f}{\sigma'_o} \right) dz}$$
 (17)

where  $\sigma'_o (= \gamma' z)$  and  $\sigma'_f (= \sigma'_o + q)$  are the initial and final effective vertical stresses respectively.

The total average degree of dissipation of excess pore pressure,  $U_p$ , is

$$U_p = 1 - \frac{1}{H} \int_0^H \left( \frac{u(z, t)}{u_o} \right) dz$$
 (18)

where  $u_o$  is the initial excess pore pressure, equal to  $q$ .

#### 4. RESULTS AND DISCUSSION

Eqn. (10) is solved numerically using the finite difference approach. The finite difference method is adopted since it is versatile and many time and depth dependent variables can easily be accommodated in the equation. Analytical solution for the present case is difficult to obtain as the governing equation is non-linear. In the finite difference method, a trial analysis is made by discretizing the clay into 10, 25, 50, 80 and 100 sub-layers and analysis done. There is no further improvement in the results when the number of sub-layers is increased beyond 50 and identical results obtained for 50, 80 and 100 sub-layers. However, in the present numerical analysis, the clay layer is divided into one hundred sub-layers and convergence ensured.

##### 4.1 Effect of Thickness of Clay

The effect of the thickness of clay is studied through a dimensionless parameter,  $q^*$  ( $= q/(\gamma' H)$ ). The variation of thickness results in variation of  $q^*$  for a given loading. Larger  $q^*$  values simulate thin layer effect (For  $H$  tending to zero (thin layer) and for a given applied stress,  $q^*$  is relatively very large).

##### 4.1.1. Effect of thickness on variation of degree of settlement with time

The variations of the average degree of settlement with time for the entire thickness,  $U_s$ , by the proposed non-linear theory, are presented in Figs. 3 and 4 for PTPB and PTIB cases respectively for different  $q^*$  and compared with the conventional one-dimensional consolidation theory by Terzaghi. The time factor,  $T_v$  is taken as  $4c_v t/H^2$  ( $H/2$  is the length of maximum drainage

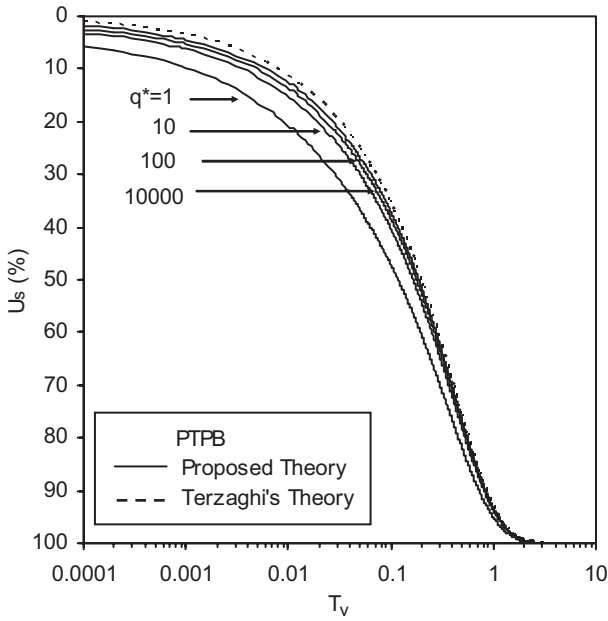


Fig. 3 Degree of settlement vs. time factor (PTPB)

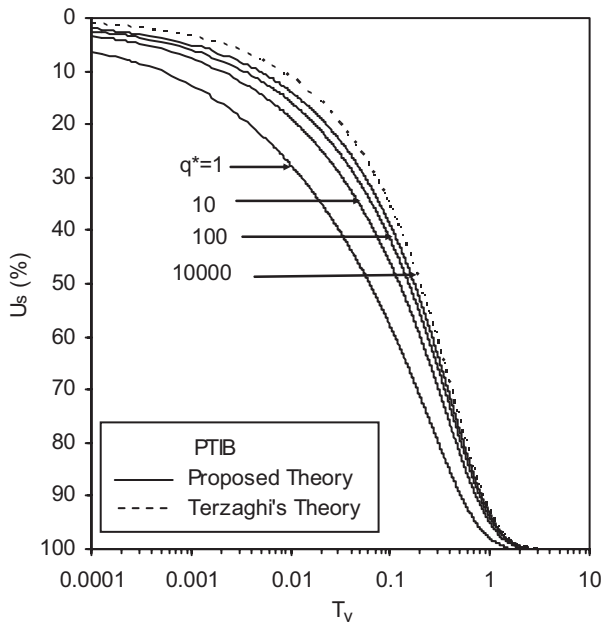


Fig. 4 Degree of settlement vs. time factor (PTIB)

path) and  $c_v t/H^2$  for PTPB and PTIB cases respectively, to compare with the conventional theory. The results from the proposed theory agree with those for the conventional thin layer theory for  $q^* [=q/(\gamma'H)] \geq 10,000$  (for  $H$  tending to zero, i.e., thin layer, the applied stress,  $q$ , relatively very large in comparison with  $\gamma'H$ ). The variation of the degree of settlement with time is relatively more in the case of thick layer of clay compared to that of a thin layer for a given load intensity. Similar observations were made by Gibson et al. (1981). The degree of settlement for a given time for a thick layer decreases with increase of  $q^*$  at all times. The

degree of settlement decreases from 60% to 51% (Fig. 3 for PTPB) and from 71% to 53% (Fig. 4 for PTIB), for  $q^*$  increasing from 1 to 10,000, at a time factor,  $T_v$  of 0.197 against a constant degree of consolidation of 50% in the conventional thin layer theory. Thus, the degree of settlement estimated from the theory of consolidation for thin layers is relatively smaller than that of the proposed non-linear theory at a given time factor.

4.1.2. Effect of thickness on time factor for a given degree of settlement

The time factors,  $T_{50}$  and  $T_{90}$  for 50% and 90% degrees of settlement from the proposed theory for various  $q^*$  values are compared with those from the linear theory in Fig. 5 for both PTPB and PTIB cases. The differences between the results from the proposed non-linear and linear theories are relatively more in the case of PTIB case compared to those for PTPB. The differences in values of  $T_{50}$  or  $T_{90}$  from non-linear and linear theories are relatively less for larger  $q^*$  values for both PTPB and PTIB cases since larger  $q^*$  values correspond to thin layer. The differences are significant only for thick layers, i.e., smaller  $q^*$  values. For example,  $T_{50}$  and  $T_{90}$  values increase respectively from 0.06 and 0.509 to 0.174 and 0.823 for PTIB for  $q^*$  increasing from 1 to 10,000 against the corresponding values of 0.197 and 0.848 in the linear theory for all  $q^*$  values.

4.1.3. Effect of thickness on variation of degree of dissipation of pore pressure with time

The excess pore pressures are computed at different depths and the average degrees of dissipation of pore pressures,  $U_p$ , for the entire thickness of clay layer are

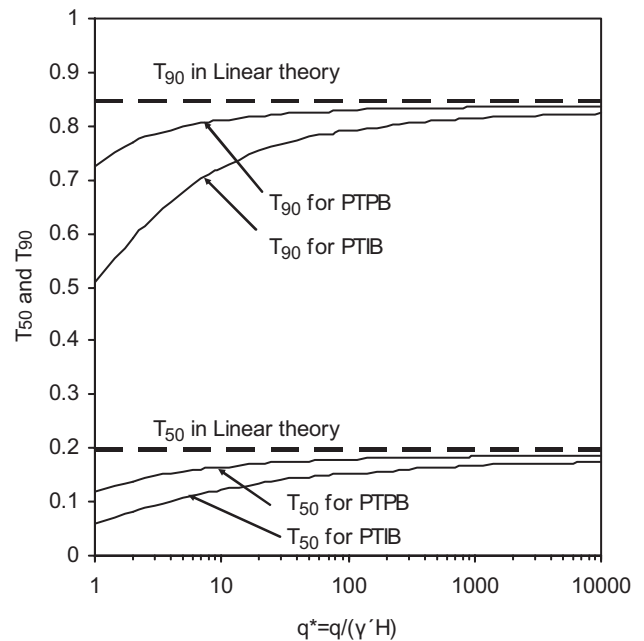


Fig. 5 Variations of  $T_{50}$  and  $T_{90}$  with  $q^*$

estimated for a given time and shown in Figs. 6 and 7 for different values of  $q^*$  for PTPB and PTIB cases respectively along with the results from conventional thin layer theory. The degree of dissipation of pore pressure from non-linear theory is slower than the degree of settlement (Davis and Raymond 1965, Gibson 1981, and Xie and Leo 2004). While the degree of settlement and the corresponding degree of dissipation of pore pressure are 51% and only 8 % for PTPB (Fig. 6) and they are 53% and 8.30% respectively for PTIB (Fig. 7),

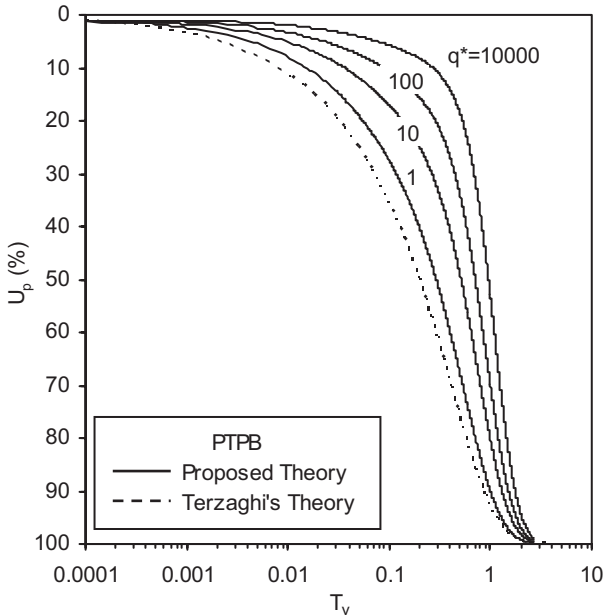


Fig. 6 Average degree of dissipation of pore pressures vs. time factor (PTPB)

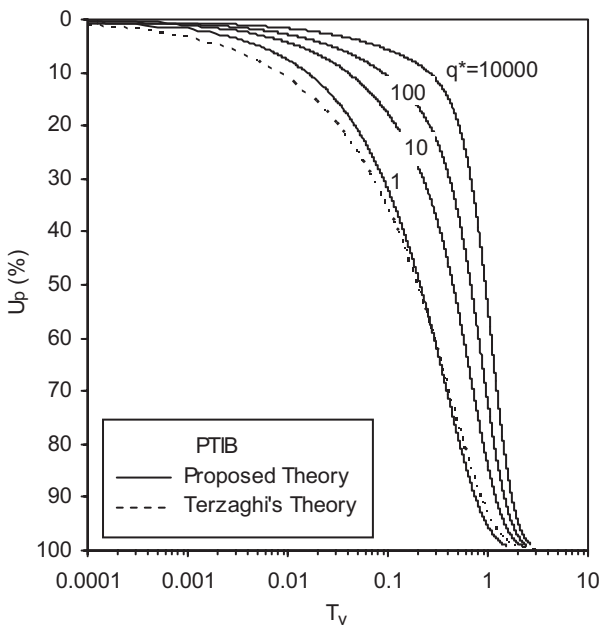


Fig. 7 Average degree of dissipation of pore pressures vs. time factor (PTIB)

for  $q^*$  of 10,000 in thick layers against a value of 50% from linear theory, at a time factor of 0.197. The degree of dissipation of pore pressure decreases with the increase of  $q^*$  as it decreases with increase of the ratio of final to initial effective stresses as established for the non-linear theory of consolidation (Raymond and Davis, 1965). Thus, the variation of degree of dissipation of pore pressures with time in the conventional thin layer theory is relatively faster or the residual pore pressures are relatively smaller in the conventional thin layer theory compared to those of the proposed theory.

#### 4.1.4 Effect of thickness on the ratio of final to initial effective stresses

Davis and Raymond (1965) have established that in the case of non-linear theory of vertical consolidation of thin layer, the degree of dissipation of excess pore pressure decreases with the increase of the ratio of final to initial effective stress ( $\sigma'_f / \sigma'_o$ ). The variation of this ratio along the depth of a clay deposit of 1 m, 5 m and 10 m are shown in Fig. 8 for an applied external load intensity of 50 kPa. The stress ratio is very large and identical for all the thicknesses near the surface of the clay deposit and it decreases sharply with depth to values of 7.95, 2.39 and 1.7 at the bottom of clay deposit of depths of 1 m, 5 m and 10 m respectively. Thus the overall average stress ratio ( $\sigma'_f / \sigma'_o$ ) for a given loading increases with the decrease in thickness of clay and this in turn slows down the degree of dissipation of pore pressure. This explains the effect of thickness of clay deposit and the reason behind the decrease of degree of dissipation of pore pressure with the increase of  $q^*$  as shown in Figs. 6 and 7.

## 4.2 Effect of Boundary Conditions

The effect of the two boundary conditions viz., (i) pervious top pervious base (PTPB) and (ii) pervious top impervious base (PTIB) is examined in terms of variations of degree of settlement and degree of dissipation of pore pressures with time in Fig. 9 and 10 respectively.

#### 4.2.1 Effect of boundary conditions on variation of degree of settlement with time

The variations of degree of settlement,  $U_s$ , with time factor,  $T_v$ , for PTPB and PTIB conditions are compared in Fig. 9. The variations of  $U_s$  with  $T_v$  are different for PTPB and PTIB conditions for thick layers for a given  $q^*$  unlike in the conventional theory where  $U_s$  is the

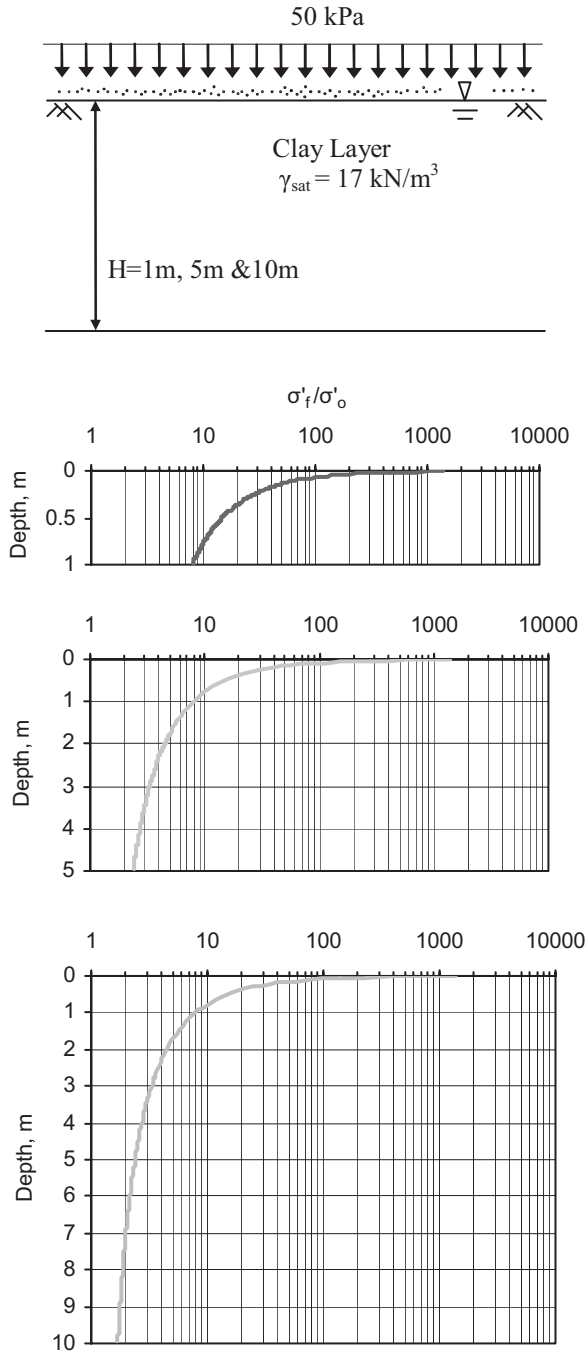


Fig. 8 Effect of thickness on  $\sigma'_f / \sigma'_o$  versus depth,  $z$

same for both the boundary conditions. The differences between results for PTPB and PTIB from the proposed theory decrease with the increase of  $q^*$  (thin layer or linear theory). The difference in  $U_s$  values is about 11% (60% and 71% for PTPB and PTIB respectively) for  $q^*=1$  at a time factor of 0.197. The difference decreases to only 2 % ( 51% and 53% for PTPB and PTIB respectively) when  $q^*$  increases to 10000.

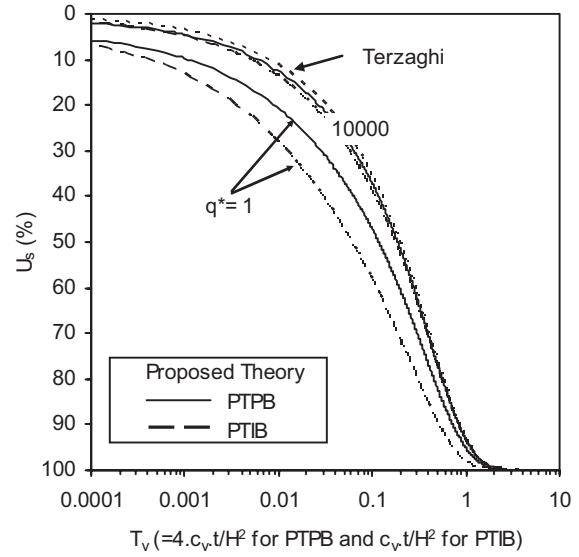


Fig. 9 Comparison of  $U_s$  vs.  $T_v$  for PTPB and PTIB

#### 4.2.2 Effect of boundary conditions on variation of degree of dissipation of pore pressure with time

The variations of degrees of dissipation of average pore pressure for the entire thickness,  $U_p$  for the cases PTPB and PTIB with time factor,  $T_v$ , are compared in Fig. 10 and found to be different like the variations of degrees of settlement of PTPB and PTIB with  $T_v$ . The difference in  $U_p$  decreases from 8% (40% and 48% for PTPB and PTIB respectively) to 0.40% (8% and 8.4% for PTPB and PTIB respectively) for  $q^*$  increasing from 1 to 10000, at a time factor of 0.197. Thus, the difference in the results ( $U_s$  and  $U_p$ ) between PTPB and PTIB at a given time factor diminishes for larger  $q^*$  values and tend to the values that correspond to those of a thin layer.

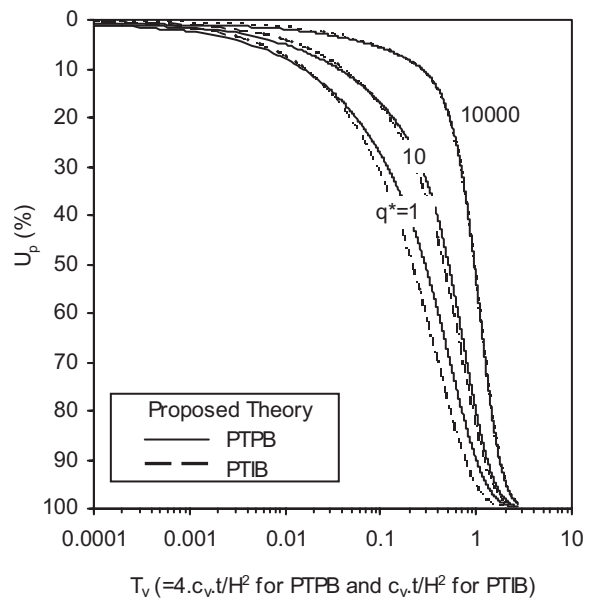


Fig. 10 Comparison of  $U_p$  vs.  $T_v$  for PTPB and PTIB

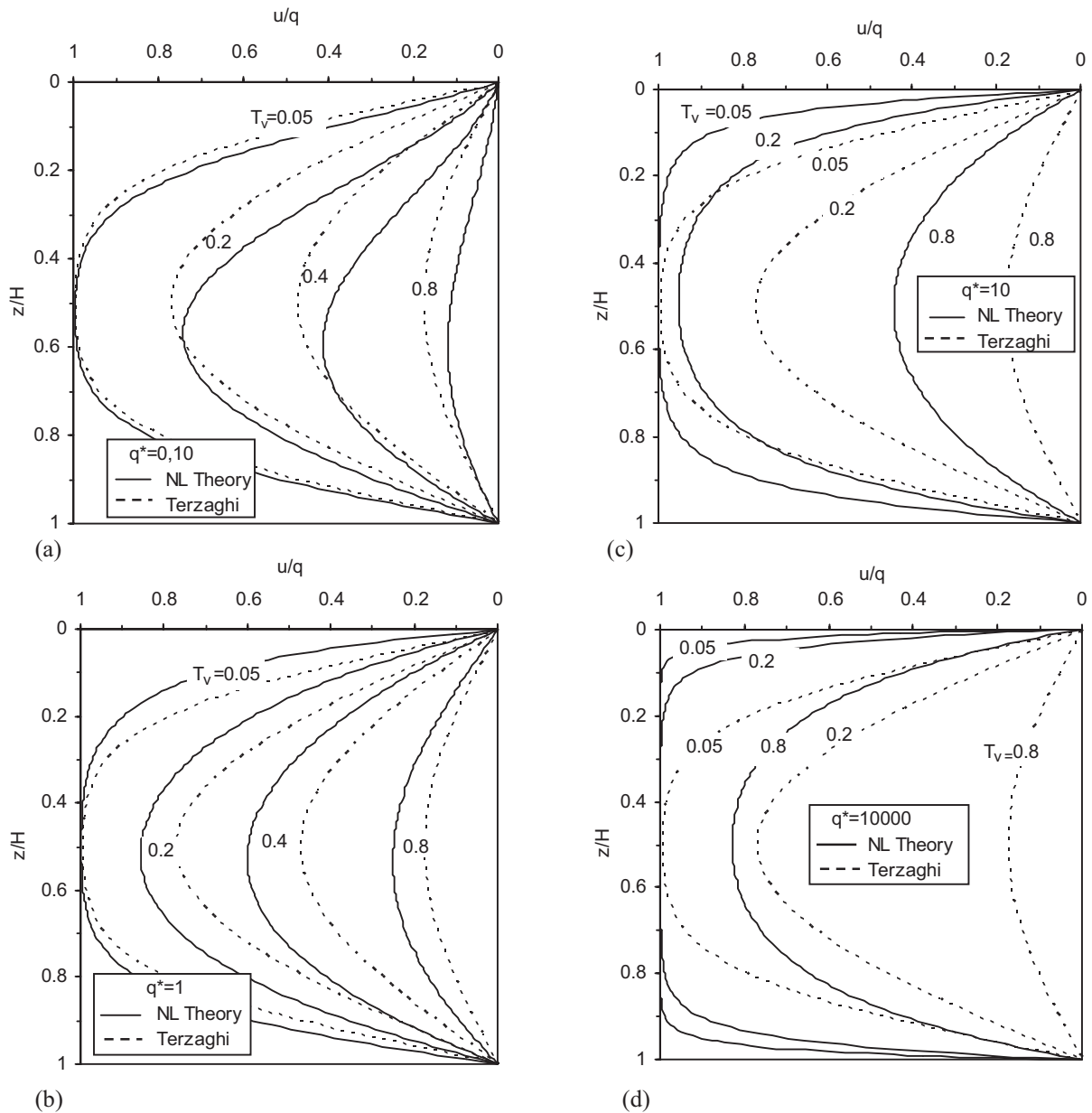


Fig. 11 Excess pore pressure isochrones for PTPB, (a)  $q^* = 0.1$ , (b)  $q^* = 1$ , (c)  $q^* = 10$  and (d)  $q^* = 10,000$

#### 4.3 Variation of Pore Pressures along Depth

The isochrones of normalized excess pore pressure,  $u/q$  for various  $q^*$  values for PTPB conditions are presented in Fig. 11 along with those from the linear theory. The excess pore pressure is relatively large or dissipation of pore pressure is relatively slow at all times according to non-linear theory of consolidation compared to those for the conventional theory. However, for  $q^* < 0.10$ , the excess pore pressure is relatively large in the case of linear theory compared to the value for the non-linear theory in the upper half of the layer. The reverse is true in the lower half of the depth.

Interestingly, the isochrones in the case of PTPB are

slightly skewed in contrast to symmetrical isochrones about the mid depth in the conventional linear theory for PTPB boundary conditions. Similar skewed isochrones are observed by Gibson et al. (1981). The residual pore pressures are 82% and 78% of  $q$ , at depths of  $0.2H$  and  $0.8H$  respectively for  $q^*$  of 10 at a time factor of 0.20. The corresponding residual pore pressure is 46% of  $q$  at the two depths from linear theory. The difference between the pore pressures of non-linear and linear theories increases with the increase of  $q^*$  at all depths as the increase of  $q^*$  results in increase of final to initial effective stress ratio which in turn influences the residual pore pressures in the non-linear theory as established by Raymond and Davis (1965).



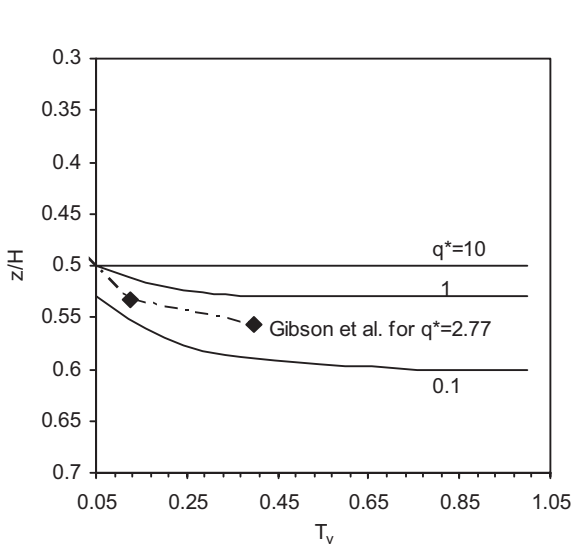
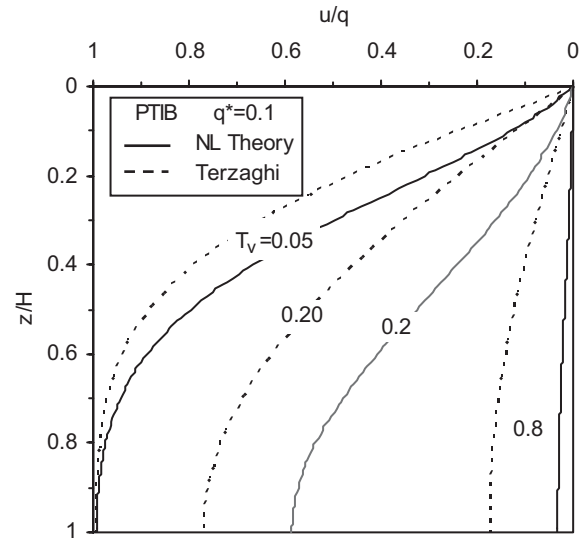


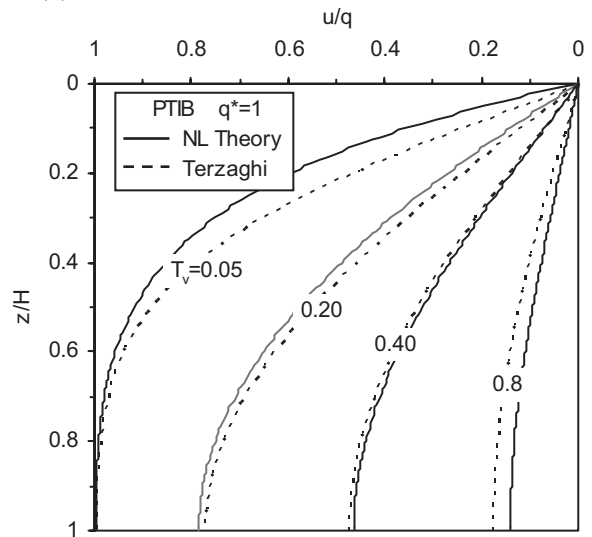
Fig. 12 Locations of maximum  $u/q$  for PTPB

The locations of the depth of maximum residual pore pressures for PTPB condition at various time factors are shown in Fig. 12. The pore pressure is maximum at mid depth for larger  $q^*$  values ( $>10$ ) while the maximum pore pressure occurs at depths more than  $0.5H$  for smaller  $q^*$  values in contrast to the results of linear theory in which case the maximum pore pressure during consolidation is always at  $0.5H$  for PTPB conditions. The presence of maximum pore pressure at depths more than  $0.5H$  is also observed in the theory proposed by Gibson et al. (1981). For an applied load intensity of 98 kPa on a submerged clayey soil of 10 m thickness ( $q^*=2.77$  for  $\gamma'$  of  $3.54 \text{ kN/m}^3$ ), the maximum residual pore pressures during consolidation are at depths more than  $0.5 H$  (Fig. 12) where  $H$  is the current total thickness at the corresponding time. The small discrepancy between the proposed theory and the theory by Gibson et al. (1981) may be due to the consideration of decrease of initial void ratio with depth and the Lagrangian and convective coordinate system adopted in the theory proposed by Gibson et al. (1981).

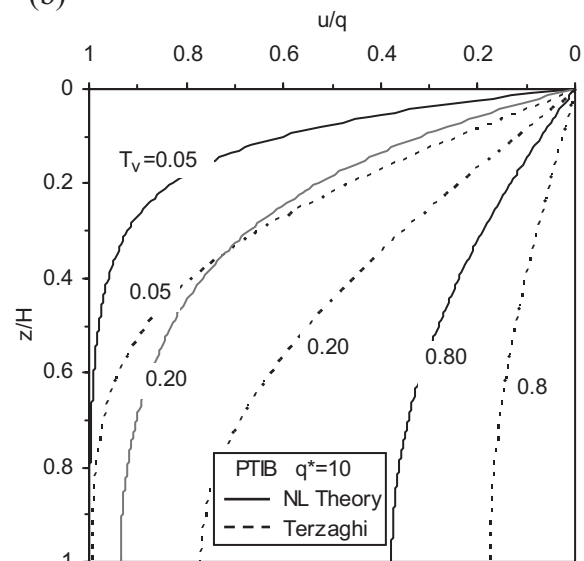
Isochrones of normalized excess pore pressure,  $u/q$  for various  $q^*$  values for PTIB condition are shown in Fig. 13 for different  $q^*$  values along with those from the linear theory. The excess pore pressures are relatively large in the conventional linear theory at all depths for small  $q^*$  values ( $<0.10$ ), compared to the values from the non-linear theory. For  $q^*=1$ , the excess pore pressures are relatively small in the linear theory in the initial stages of consolidation ( $T_v < 0.4$ ) and more in the later stages where as, for  $q^*=10$ , the excess pore pressures are relatively small in the linear theory at all stages of consolidation as the dissipation of pore pressure is slow according to the non-linear theory at high stress ratio.



(a)



(b)



(c)

Fig. 13 Excess pore pressure isochrones for PTIB, (a)  $q^* = 0.1$ , (b)  $q^* = 1.0$  and (c)  $q^* = 10$ .

5. EXAMPLE CALCULATION

The results of the proposed theory are examined by comparing the results for a typical problem from the present approach with those from the conventional linear theory of Terzaghi and the non-linear theory developed by Gibson et al. (1981). A clay layer of 10 m thick pervious at both the top and the bottom boundaries is considered. The comparison uses the soil properties (Osaka Harbor mud) shown in Table 2, used by Gibson et al. (1981).

As part of land reclamation, surcharge loadings,  $q$ , of intensities of 9.8 kPa and 98 kPa are proposed for consolidation of the clay layer. The corresponding non-dimensional loading parameters,  $q^*$  works out to be 0.277 and 2.77 respectively. The results of analysis using the proposed theory are presented in Figs. 14, 15 and 16.

Fig. 14 shows the variation of consolidation settlement with time based on the proposed theory and the conventional linear theory. The ultimate settlement is calculated as per the following formula.

$$S_f = H_o \frac{C_c}{(1 + e_o)} \log_{10} \left( \frac{\sigma'_f}{\sigma'_o} \right) \quad (19)$$

The clay layer is divided into 100 thin layers and for each layer,  $\sigma'_o$  and  $\sigma'_f$  are calculated and the ultimate consolidation settlement is worked out. The sum of the ultimate consolidation settlements of these layers is taken as the total consolidation settlement of the clay deposit. For all the layers, the ultimate settlement is computed considering the initial void ratios equal to the average void ratios computed by Gibson et al. (1981) for various loads. In Gibson’s paper, the initial void ratio is taken as 3.83 at the top of clay deposit corresponding to the water content of 140% and as 1.97 at the bottom corresponding to an initial in-situ effective stress of 35 kPa. The average void ratios are taken as 2.11 and 1.73 for applied loads of 9.8 kPa and 98 kPa respectively corresponding to the average effective stresses of 23 kPa and 67 kPa respectively. These average stresses are taken equal to half of the ultimate effective stress at the bottom of clay deposit, as per Gibson et al (1981).

Table 2: Soil properties – Osaka Harbor Mud

Liquid limit ( $w_L$ )	90%
Plastic Limit ( $w_p$ )	30%
Water content	140%
Compression index ( $C_c$ )	0.8
Coefficient of consolidation ( $c_v$ )	$1.944 \times 10^{-3} \text{ m}^2/\text{day}$
Total unit weight of soil ( $\gamma_{sat}$ )	$13.33 \text{ kN/m}^3$
Specific gravity of soil ( $G_s$ )	2.74

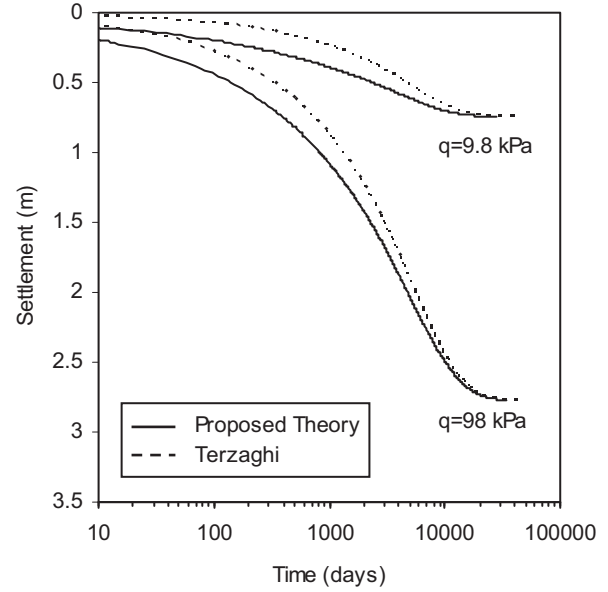


Fig. 14 Comparison of settlement variations from the proposed and linear theories

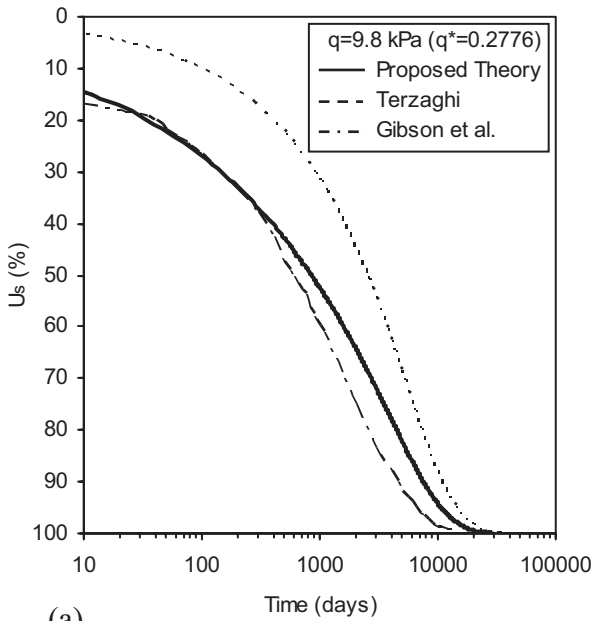
The ultimate consolidation settlement for a given loading is the same from both the proposed and the conventional linear theories as the formulation for calculation of the ultimate settlement is the same in both the theories. However, the settlements at different times are different as the variation of degree of settlement is dependent on the non-dimensional loading parameter,  $q^*$  (Fig. 3) in the present theory. Thus the settlements at intermediate times are relatively small from the linear theory, compared to those from the proposed theory. For example, settlements are 0.87 m and 1.08 m at a time of 1,000 days for a load of 98 kPa as per the linear and non-linear theories respectively.

The variation of degree of settlement with time based on the proposed theory is compared with that of non-linear theory proposed by Gibson et al. (1981) based on Lagrangian coordinate system and the conventional linear theory in Figs. 15 (a) and 15(b) for loads of 9.8 kPa and 98 kPa respectively. The proposed theory is in agreement with non-linear theory proposed by Gibson et al. (1981) in the initial stages of consolidation up to about 200 days. The results of the proposed theory shift away from those proposed by Gibson et al. (1981) as consolidation proceeds and tend to approach those from the linear theory. This variation is due to the consideration of decrease of initial void ratio with depth, finite strain coefficient of consolidation and the Lagrangian coordinate system adopted in the theory proposed by Gibson et al. (1981), where in the degree of settlement is computed as

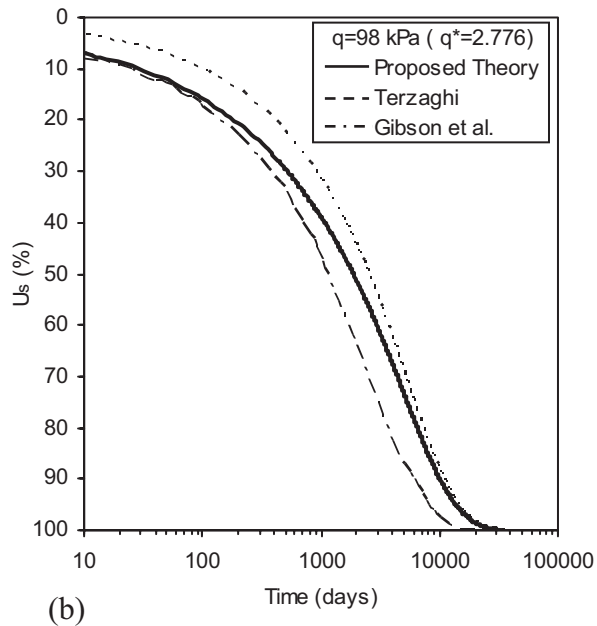
$$U(t) = \frac{S(t)}{S(\infty)} = \frac{\int_0^l [e(z,0) - e(z,t)] dz}{\int_0^l [e(z,0) - e(z,\infty)] dz} \quad (20)$$

where  $l$  is the total depth of solids taken as 2.73 m for total depth of clay of 10 m. The finite strain coefficient of consolidation ( $g$ ) is taken as

$$g = \frac{c_v}{(1+e)^2} \quad (21)$$



(a)



(b)

Fig. 15 Degrees of settlement from the proposed, linear and Gibson’s theories, (a)  $q=9.8$  kPa and (b)  $q=98$  kPa

Therefore, the results of the proposed theory lie between those from the non-linear theory proposed by Gibson et al. (1981) and the conventional linear theory.

The variations of consolidation settlement with time based on the proposed theory and the non-linear theory proposed by Gibson et al. (1981) are compared in Fig. 16. For light loads (say 9.8 kPa) the variations of consolidation settlement with time based on the proposed and the non-linear theories proposed by Gibson et al. (1981) are in good agreement. However, for larger loads, the difference between the two theories is somewhat considerable and the magnitude of difference increases with the increase of load. The ultimate consolidation settlements is 2.77 m as per the proposed theory against 3.09 m from the theory proposed by Gibson et al. (1981) for a load of 98 kPa. Nevertheless, the proposed theory provides a simplified approach for the analysis of thick layer consolidation based on the non-linear approach.

The limitations in the proposed method include constant thickness of clay during consolidation (infinitesimal strain) and constant initial void ratio along the depth. However in practice, the thickness of clay layer decreases during consolidation and the initial void ratio may decrease slightly with the depth of a thick clay deposit.

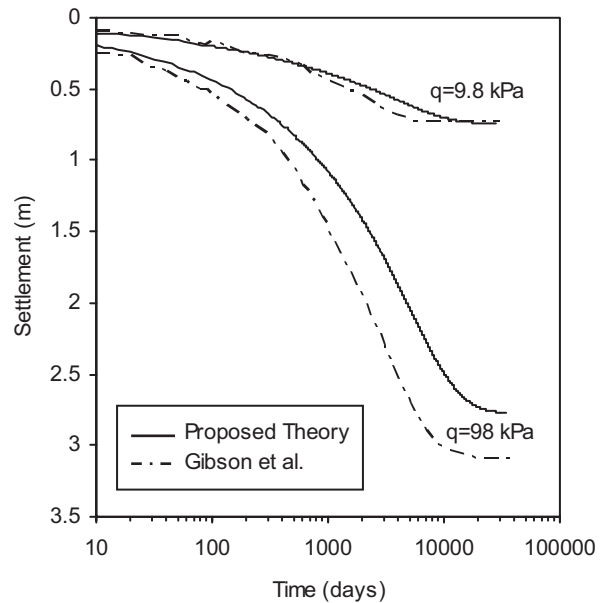


Fig. 16 Settlement versus time from the proposed and Gibson’s theories

## 6. CONCLUSIONS

A simple non-linear theory of one-dimensional consolidation for thick clay layer is developed considering linear void ratio - log effective stress relationship, constant value of coefficient of consolidation, approximately constant initial void ratio with depth but accounting for the variation of initial effective stress with depth and PTPB (pervious top and pervious base) and PTIB (pervious top and impervious base) boundaries. The degree of settlement is relatively large while the degree of dissipation of excess pore pressure is relatively small at a given time in the case of thick layer of clay compared to those for a thin layer. The variations of degree of settlement with time factor for PTPB and PTIB conditions are different for thick clay layer unlike in the conventional linear theory wherein the variations are the same for both conditions. The isochrones in the case of a thick layer with PTPB condition are slightly skewed in contrast to symmetrical isochrones of thin layer theory. The variations of degree of settlement and degree of dissipation of pore pressures are sensitive to the magnitude of loading in the case of a thick layer while these are independent of loading from the thin layer theory. The results compare well with those from Gibson et al. (1981) but for the consideration of variation of initial void ratio with depth considered by the latter.

## REFERENCES

- Abbasi, N., Rahimi, H., Javadi, A.A. and Fakher, A. (2007). Finite difference approach for consolidation with variable compressibility and permeability. *Computers and Geotechnics*. 34(1): 41-52.
- Chen, R.P., Zhou, W.H., Wang, H.Z. and Chen, Y.M. (2005). One-dimensional nonlinear consolidation of multi-layered soil by differential quadrature method. *Computers and Geotechnics*. 32(5):358-369.
- Choa, V. (1995). Compression and consolidation of clayey soils, Yoshikuni and Kusakabe (eds), Balkema, Rotterdam: 1005-1017.
- Conte, E. and Troncone, A. (2006). One-dimensional consolidation under general time-dependent loading. *Canadian Geotech. J.* 43(11): 1107-1116.
- Conte, E. and Troncone, A. (2007). Nonlinear consolidation of thin layers subjected to time-dependent loading. *Canadian Geotech. J.* 44(6): 717-725.
- Davis, E.H. and Raymond, G.P. (1965). A Non-linear theory of consolidation. *Geotechnique*. 15(2): 161-173.
- Gibson, R.E., Schiffman, R.L. and Cargill, K.W. (1981). The theory of one-dimensional consolidation of saturated clays: II. Finite nonlinear consolidation of thick homogeneous layers. *Canadian Geotechnical Journal*. 18(2): 280-293.
- Lee, P.K.K., Xie, K.H. and Cheung, Y.K. (1992). A study on one-dimensional consolidation of layered systems. *Int. J. Numerical and Analytical methods in Geomechanics*. 16: 815-831.
- Lekha, K.R., Krishnaswamy, N.R. and Basak, P. (2003). Consolidation of clays for variable permeability and compressibility. *J. Geotech. and Geoenviron. Engrg.* 129(11): 1001-1009.
- Richart, F.E. (1957). Review of the theories for sand drains. *Transactions of ASCE*. 124(2999): 709-736.
- Vaid, Y. P. (1985). Constant rate of loading nonlinear consolidation. *Soils and Foundation*. 25 (1): 105-108.
- Walker, R., Indraratna, B. and Sivakugan, N. (2009). Vertical and radial consolidation analysis of multilayered soil using the spectral method. *J. Geotech. Geoenviron. Eng., ASCE*. 135(5): 657-663.
- Xie, K.H. and Leo, C.J. (2004). Analytical solutions of one-dimensional large strain consolidation of saturated and homogeneous clays. *Computers and Geotechnics*. 31: 301-314.
- Xie, K.H., Xie, X.Y. and Jiang, W. (2002). A study on one-dimensional nonlinear consolidation of double-layered soil. *Computers and Geotechnics*. 29: 151-168.
- Xie, K.H., Wang, K., Wang, Y.L. and Li, C.X. (2010). Analytical solution for one-dimensional consolidation of clayey soils with a threshold gradient. *Computers and Geotechnics*. 37 (4): 487-493.
- Zhuang, Y. C., Xie, K. H. and Li, X.B. (2005). Nonlinear analysis of consolidation with variable compressibility and permeability. *Journal of Zhejiang University (Engineering Science)*. 6A (3): 181-187.