Experimental Investigation for Obtaining the Reflected and Transmitted Waves on Rubble-Mound Breakwater:

A model simulation study of the Ariake seashore

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Summary

The irregular wave reflection and transmission over porous submerged breakwater was experimentally predicted. The experiments were conducted in the laboratory in which sea waves were simulated using a 1/12 scale model of the Ariake Reclamation Dike. The dimensional analysis of all parameters of the wave flume followed from the above-mentioned scale model was done by Froud similitude.

An extensive series of physical tests were conducted which included transmission wave availability in which the overtopping wave height on the dike is not greater than the permitted height. Three types of breakwater height were constructed in and analyzed.

The reflection coefficients of wave by comparing between the spectral measurements and Healy's method were all in good agreement. The transmissions coefficient are presented as an alternative reference for construction the rubble mound breakwater under field conditions like the Ariake seashore.

Introduction

Submerged breakwaters are frequently used in the shoreline or harbor as a protection structure. Low crested rubble-mound breakwaters may also become submerged after being damaged by the waves attack. The submerged breakwaters are many being constructed because of their low cost, aesthetics, and effectiveness in protecting from high waves breaking not only for harbor and shoreline but also a structure that can develop the function of seawall (dike).

Many experiments have been conducted related to wave reflection and transmission over a porous submerged breakwater. Nevertheless, they can be more detail examined based on the field experiments or laboratory simulation that the models or research material are resembled to the real situation.

In this paper, the model size of dike made in the laboratory simulation of sea waves was 1/12 times of the Reclamation Dike in Ariake seashore. The dimensional analysis of all parameters followed from the above-mentioned scale model was done by Froud

similitude.

Since the annual report of typhoon attack from Saga University Observation System results, the significant wave heights in the near–shore zone were still extremely height [5], or in other words, the waves overtopping occurred on the ariake reclamation dike were still slightly big which consequently damage the reclamation dike and/or to flood in the landward side, thus the Ariake seashore was obtained as an experimental object of this research. Table 1 is a prototype and model wave height in which the prototype data was only taken the four highest values from the report of Saga University Observation System[5].

Table. 1. Prototype and model of occurred wave height by typhoons.

Typhoon no.	Occurred date	$H_{1/3}$(cm) Prot.	H_{max}(cm) Prot.	$H_{1/3}$(cm) Model	H_{max}(cm) Model
9117	1991.9.14	236	347	19.66	28.91
9109	1991.7.29	160	279	13.33	23.25
8513	1985.8.31	230	326	19.16	27.16
8410	1984.8.21	161	296	13.41	24.66

One of the solution for reducing the wave overtopping which is purposed to develop the function of the reclamation dike is to arrange the submerged rubble mound breakwater explained above, away from the reclamation dike (Seawall). Whenever this structure is located, then the damaging waves can be minimized become waves transmission. To determine the optimum values of coefficient transmission, the incident waves and reflection waves are certainly must be considered in the experiments.

Experimental Setup

To carry out the aim of this research, experiments were conducted in the horizontal bed of wave flume at Saga University's Water Supply and Management Engineering Laboratory, Japan. The wave flume is 80 cm wide, 26 m long, and 100 cm deep. In its half part is set–up a glass side–wall. At one end of the wave flume, a random wave generator was installed while a model of Ariake Sea Reclamation dike was set in the other end (Fig.1). The sea depth in front of the dike was determined as 43.5 cm. This value was calculated by applying the analysis dimensional based on the maximum actual sea-depth (T.P. + 3.8, [9]) has been occurred when the typhoon came. All tests were performed with irregular waves made by the JONSWAP spectrum and measured by means of three wave-gage installations on the wave flume. There were 135 data set has been recorded where each record was digitized by an A–D converter at sampling interval of 0.05 sec. The number of sampling points is 3100. The sampling data was initiated only after the reflections had stabilized.

The results of the digitization data profile correspond to range of 1.0–1.3 sec of significant period (T_s). They are similar with the prototype significant period about 3.46–4.50 sec. Moreover, the expected incident and transmitted wave heights were in the range

of 11-15 cm and 6-14 cm respectively, or similar with the prototype wave heights about 1.32- 1,8m and 0.72-1.68m.
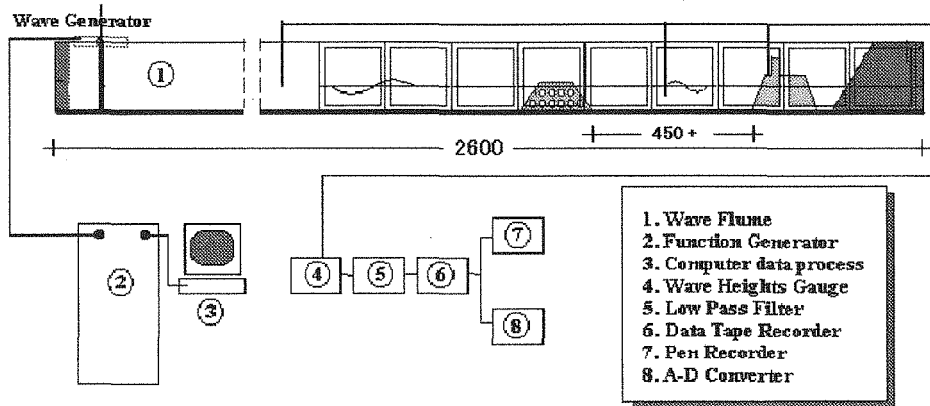

Fig. 1.   Test equipment.

A model of Ariake reclamation dike (Fig.2b) has made and locates in the one end before the absorber beach while the model of rubble-mound breakwater covered the artificial tetrapods (Fig.2a) was placed by turn every 50 cm from 5.00 m to 7.00 m in front of the dike and arranged for 3 crest height as shown in Table 2.

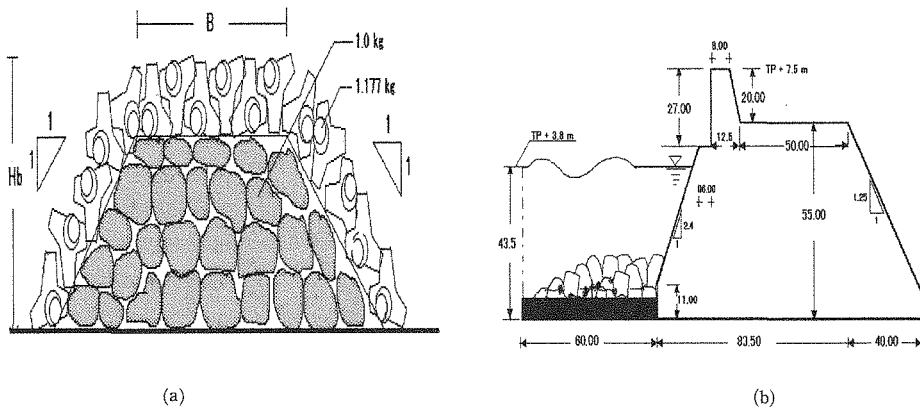(a)   (b)

Fig. 2.   A model of rubble-mound breakwater and Ariake reclamation dike.

Since the construction of rubble-mound breakwaters in relatively deep water and wave-exposed locations necessitates the use of heavy protective armor units which can not be obtained economically from a quarry mine in observation field, the experiments used tetrapod concrete armor units in place of the hard-to-obtain stone.

The Hudson coefficient, K_D was taken as 6, which is proposed for non-breaking waves at the structure head.   The calculations of the breakwater design criteria by using the Hudson formula are presented in the Table 2.

The Hudson formula to determine the weight of armor units for rubble-mound structures with the criteria of wave height as no-damage conditions and no-overtopping

conditions is currently used:

$$W = \frac{\gamma_r H^3}{K_D(S_r - 1)^3 \cot \alpha} \tag{1}$$

As can bee seen in Table 2, the weight per unit of tetrapod is 1.177 kg. This is similar with the prototype weight that can be placed in the actual situation around 2000 kg/unit for the dimensional scale of 1/12.

The thickness (r) of the cover layers and the number of armor units (N_r) are respectively found from Eqs. (2) and (3).

$$r = nk_\triangle \left(\frac{W}{\gamma_r}\right) = nk_\triangle V^{1/3} \tag{2}$$

$$N_r = Ank_\triangle \left(1 - \frac{P}{100}\right)\left(\frac{\gamma_r}{W}\right)^{2/3} \tag{3}$$

in which W : weight of protective armor units; γ_r : specific weight of armor units; S_r : specific gravity of armor units : γ_r/γ_w ; $\alpha =$ angle of breakwater slope; H : design wave height at the structure site; and K_D : damage coefficient, depending primarily on block shape, r : the thickness of n layers of armor units of weight W and of specific weight γ_r; and $k_\triangle$: a layer coefficient which measures the packing density of the units of volume V, P : the average porosity of the layer, as a percentage. Values of $k_\triangle$ and P can be seen in the reference no. 1.

Table. 2. The dimension of rubble-mound breakwater's model parameters

Type	K_D	n	$B(cm)$	$H_b(cm)$	$W(kg/unit)$	$A(m^2)$	$N_r(unit)$
Crest H.A	6.0	1	50	23	1.177	0.92	76
B	6.0	1	65	35	1.177	1.31	108
C	6.0	1	80	43.5	1.177	1.62	134

1. Wave Height Statistics

Many studies have advanced a statistical approach for the analysis and prediction of sea wave behavior[5]. For the present investigation, the most significant analysis provided by Longuet-Higgins[6] in which the results of Rice[10] were applied to deduce the probability distribution of wave heights. By assuming that the wave frequency spectrum consists of a relatively narrow band of frequencies, and that the sea waves are the superposition of many sinusoidal components of about the same frequency but of random phase, Longuet-Higgins determined that the probability distribution of wave heights $p(H)$ must be a Rayleigh distribution; that is:

$$p(H)dH = \frac{\pi}{2}\frac{H}{\mu^2}\exp\left[-\frac{\pi}{4}\left(\frac{H}{\mu}\right)^2\right]dH \tag{4}$$

in which H: the wave height, and μ : mean wave height. By using the properties of the first and second moments of the probability distribution, a relation between the mean wave height μ and root-mean-square wave heights H_{rms} is obtained

$$H_{rms} = \left(\frac{2}{\sqrt{\pi}}\right)\mu = 1.129\mu \tag{5}$$

Furthermore, Longuet-Higgins showed that

$$H_{1/n} = H_{rms}[\sqrt{1nn} + (n/2)\sqrt{\pi}(1 - erf\sqrt{1nn})] \tag{6}$$

in which $H_{1/n}$ = the mean height of the highest $1/n$ (fraction) of the waves.
From Eq. (3) it follows that, for $n=3$

$$H_{1/3} = 1.598\mu \text{ And for } n=10, \ H_{1/10} = 2.032\mu \tag{7}$$

in which $H_{1/3}$: the significant wave height, and $H_{1/10}$: the mean height of the highest one-third of the waves.

The term "significant wave height," $H_{1/3}$ or Hs mentioned above has traditionally been defined as the average height of the highest one-third of the individual waves as the average height of the highest one-third of the individual waves in a record as suggested by Munk[7]. In this experiment, the significant wave height, $H_{1/3}$ was estimated from wave gauge records by simple procedures. In deep water, a useful estimate of significant height that is fundamentally related to wave energy is defined as

$$H_{mo} = 4\sigma \tag{8}$$

in which H_{mo} : an internationally recommended notation means[10]; and $\sigma =$ the standard deviation of sea surface elevations.

Experimental results and calculations based on the Rayleigh distribution function show that when wave shapes are not severely deformed by shallow-water depth or high wave steepness, the following approximation can be used[1].

$$H_{mo} = H_{1/3} \tag{9}$$

However, it has been suggested that when H_{mo} differs from H_s, H_{mo} cannot be used directly to estimate height statistics[8].

2. Incident and Reflected Wave

All the problems related to the phenomenon of wave reflection require the data reflection coefficient, or the ratio of reflected to incident wave heights. In the problems involving the dissipation of wave energy that are applied for most of actual structures, the reflection coefficient cannot be predicted analytically but must be measured experimentally.

E.P.D. Mansard and E.R. Funke[3] have expanded the method to measure reflected wave from the beach structure in the laboratory simulation of sea waves.

This method employs a combination of fastest spectral analysis which known as FFT and Least Square Error for separating the incident and reflected spectra which then to be interpreted into the incident and reflected waves.

This method assumes the waves are traveling in a channel in a longitudinal direction and reflections from some arbitrary structure or beach are traveling in the opposite direction. It is assumed also that it is possible to measure simultaneously the linear superpositions of these waves at m points $p=1, 2, 3$ to m, which are in reasonable proximity to each other and are on a line paralel to the direction of wave propagation.

The wave field might be better described by a sum of sinusoidal terms, thus for a profile observed at any one of the wave gauge positions may be written harmonically related Fourier components as follow,

$$\eta_p(t)=\sum_{k=1}^{N} A_{p,k} \cdot \sin\left(\frac{2\pi kt}{T}+\alpha_{p,k}\right) \tag{10}$$

where $A_{p,k}$ is the Fourier coefficient for frequency k/T, T : the length of the wave profile which is being observed, $\alpha_{p,k}$ = the phase relative to the time origin of the record, N : an upper limit of summation which depends on the maximum significant frequency component in the series. The Fourier coefficients and their phases are obtained from a Fourier transform of the function, $\eta_p(t)$, $0 \leq t \leq T$ and are given in polar form as $B_{p,k}=A_{p,k}.e^{i\alpha pk}$, k or in rectangular form as $B_{p,k}=[A_{p,k}.\cos(\alpha_{p,k})+i.A_{p,k}.\sin(\alpha_{p,k})]$.

Since the spacing between the various probes is known and since it is established that (except for lock harmonics) individual frequency components in a composite wave train travel at their own celerity[3], it is possible to calculate the phase relationships between the wave trains as observed by each of the three probes. Thus, the general equation for a progressive wave can be written

$$\eta_x(t)=\sum_{k=1}^{N} C_k \cdot \sin\left(-\frac{2\pi kt}{T}+\frac{2\pi x}{L_k}+\theta_k\right) \tag{11}$$

where θ_k is some arbitrary phase related to the space and time origin of the function, x is a space variable measured from the space origin of the function in a direction of wave propagation, L_k is the wavelength of frequency k/T. The observation of wave activity made at point p can now be stated in terms of a summation of, a). an incident wave $C_{I,k}$, b). a reflected wave $C_{R,k}$, c). a noise signal which may be caused due to cross-nodal activity, locked harmonics, non-linear interactions, measurement error.

If the $X1$ is the distance from the wave source to the wave gauge at $p=1$ and XR1 is the distance from the wave gauge at $p=1$ to the reflecting structure, then the wave profile as observed at the probe may be written as:

$$\eta_{p1}(t)=\sum_{k=1}^{N} C_{I,k} \cdot \sin\left(-\frac{2\pi kt}{T}+\frac{2\pi(X1+X1P)}{L_k}+\theta_k\right)$$

$$\sum_{k=1}^{N} C_{R,k} \cdot \sin\left(-\frac{2\pi kt}{T}+\frac{2\pi(X1+2X1P1-X1P)}{L_k}+\theta_k+\phi_k\right)+\Omega_1(t) \tag{12}$$

and its explanation can be seen in Fig.3 as follow.
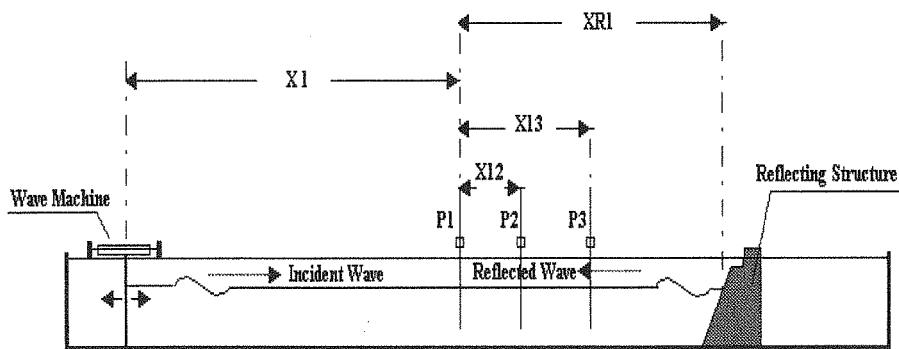

Fig. 3. Set-up for wave reflection measurement.

where $\Omega_1(t)$ is the cumulative effect of all the corrupting signals at probe $p=1$ and is a phase change due to the reflecting structure. The second probe at $p=2$, which is displaced

by a distance $X12$ from the probe at $p=1$ in the direction of incident wave propagation (see Fig. 3), will record a similar wave profile as Eq.(12) except the phase angles will now be

$$\left(-\frac{2\pi kt}{T}+\frac{2\pi(X1+X12)}{L_k}+\theta_k\right) \tag{13}$$

for the incident wave and

$$\left(-\frac{2\pi kt}{T}+\frac{2\pi(X1+2X1P1-X12)}{L_k}+\theta_k+\phi_k\right) \tag{14}$$

for the reflected wave. A similar argument will define the wave angles for other probe positions that are displaced by distances $X1P$ from the first probe.

After some arrangement include applying the least square error method[3] to the eqs. (12), (13) and (14), finally yields the equations used in this measurements i.e.:

$$Z_{I,k}=\frac{1}{D_k}(B_{1,k}(R1+iQ1)+B_{2,k}(R2+iQ2)+B_{3,k}(R3+iQ3)) \tag{15}$$

and

$$Z_{R,k}=\frac{1}{D_k}(B_{1,k}(R1-iQ1)+B_{2,k}(R2-iQ2)+B_{3,k}(R3-iQ3)) \tag{13}$$

in which

$$D_k=2.(\sin^2\beta_k+\sin^2\gamma_k+\sin^2(\gamma_k-\beta_k); \quad R1_k=\sin^2\beta_k+\sin^2\gamma_k$$
$$Q1_k=\sin\beta_k\cos\beta_k+\sin\gamma_k\cos\gamma_k \qquad ; R2_k=\sin\gamma_k+\sin(\gamma_k-\beta_k)$$
$$Q2_k=\sin\gamma_k\cos(\gamma_k-\beta_k)-2\sin\beta_k \qquad ; R3_k=\sin\beta_k+\sin(\gamma_k-\beta_k) \tag{16}$$
$$Q3_k=\sin\beta_k\cos(\gamma_k-\beta_k)-2\sin\gamma_k$$

From $Z_{I,k}$ and $Z_{R,k}$ compute the spectrum densities $S_{I,k}$ and $S_{R,k}$ by

$$S_{I,k}=|Z_{I,k}|^2/(2\Delta f) ; S_{R,k}=|Z_{R,k}|^2/(2\Delta f) \tag{17}$$

The reflection coefficient function is then evaluated from

$$K_{R,k}=|Z_{R,k}|/|Z_{I,k}| \tag{18}$$

while the average reflection coefficient is evaluated from

$$K_R=\frac{H_R}{H_I}=\sqrt{m_{0,r}}/\sqrt{m_{0,i}} \tag{19}$$

in which

$$m_{0,i}=\int_{f_{min}}^{f_{max}}S_i(f).df ; m_{0,r}=\int_{f_{min}}^{f_{max}}S_r(f).df \tag{20}$$

The incident and reflected wave height can be finally obtained by using the following equation.

$$H_I=\frac{1}{\sqrt{1+K_R^2}}H_s \text{ And } H_R=\frac{K_R}{\sqrt{1+K_R^2}}H_s \tag{21}$$

3. Wave Transmission

When waves strike a breakwater, wave energy will be transmitted after they have been firstly reflected and dissipated at the structure.

The transmission coefficient outcomes from a significant amount of wave transmission for submerged breakwaters done in this research are calculated using the following equation.

$$K_T=\frac{H_T}{H_I} \tag{22}$$

Experimental Results

Longuet–Higgins[6] concluded that the wave heights should have a Rayleigh probability distribution under the conditions already described. When plotted on a Rayleigh probability coordinate axis, the cumulative distribution of wave heights should lie along a straight line if the conclusion is valid. For each case, a straight line has been visually fitted to demonstrate a graphical test of that conclusion.

In most cases, an assumption that the wave heights are Rayleigh distributed appears to be quite reliable. The derivation from the trend is probably unexplainable for a few cases. However, for engineering purposes it seems appropriate to consider the usefulness of the basic assumption to be confirmed. Two example results can be seen in Fig.4 and 5.
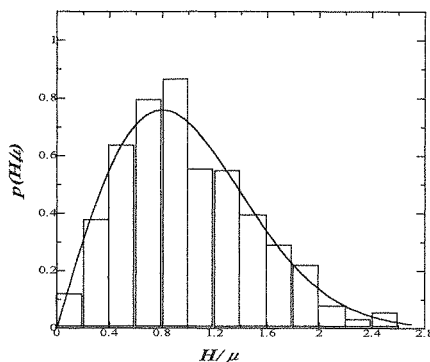
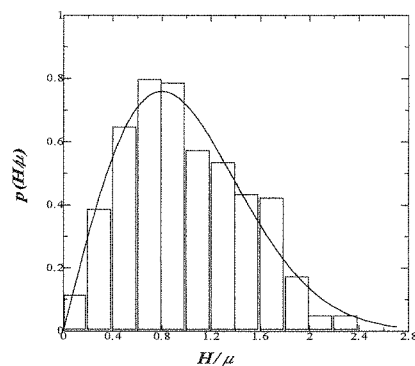Fig. 4. Frequency distribution of H/μ of incident wave, B-1.

Fig. 5. Frequency distribution of H/μ of transmitted wave, B-1.

It is also possible to test the basic assumption through the relationships of various wave-height parameters. These are usually computed as ratios with the mean wave height. Three such parameters are listed for their subsequent use in testing the experimental data. They are the ratios given by Eqs. (5) and (7): 1) Mean of the highest one-tenth waves to mean wave height; 2) significant wave height tp mean wave height; and 3) Root-mean square wave height to mean wave height. Figs. 6,7 and 8 show the comparison of data between experiment and theoretical relation.

As explained in the previous part ideally, when waves strike a breakwater, wave energy will be reflected from, dissipated on, or transmitted through or over the structure. The way incident and wave energy is partitioned between reflection, dissipation, and transmission depends on incident wave characteristics (period, height, and water depth), and the geometry of the structure (slope, crest elevation relative to SWL, and crest width). Briefly, it can be said that all structure includes breakwaters and dikes should reflect any wave energy approaching to them. This opinion suggests for considering the occurrence-reflected wave of every experiment is being done on the coastal structures. By using the method that explained in the previous part and applied all the mentioned equations by employing some computer programs, the reflection coefficients were measured.

The measured reflection coefficients in seaside were about 25–35 %. In the transmis-
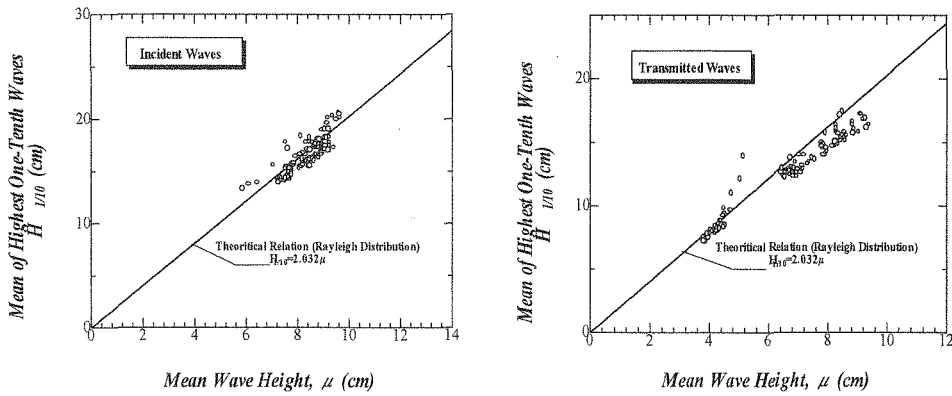
Fig. 6. Graph of mean of the highest one-tenth of the waves vs
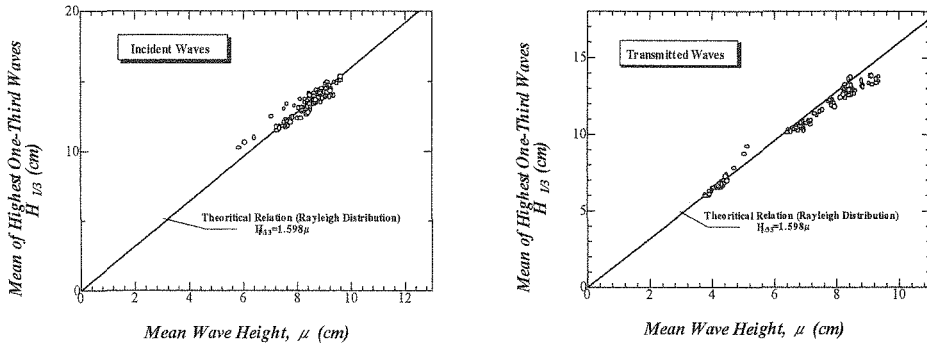mean wave height of incident and transmitted waves.

Fig. 7. Graph of mean of the highest one-third of the waves vs
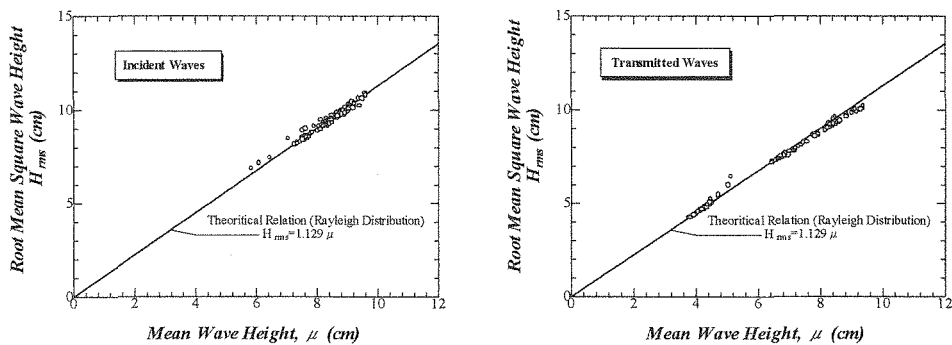mean wave height of incident and transmitted waves.

Fig. 8. Graph of root-mean-square wave height vs
mean wave height of incident and transmitted waves.

sion side these coefficients were about 50-65 %. These slightly big values occurred because of the reflected structure was made by the concrete that possibly reflects until 100 % of oncoming waves energy. The figures represented the measurement of incident and reflection waves are given as an example in the Fig.9 and 10.

In the computation of incident and reflected waves, the significant height that is fundamentally related to wave energy (H_{mo}) was also been calculated. This H_{mo} was then compared to the one-third significant wave height for confirming the reliability data were used in the analysis (see Fig. 11). The comparison result shows that the H_{mo} values smaller than the $H_{1/3}$. The value of significant wave heights used in this experiment was $H_{1/3}$ due to the different between the results of H_{mo} and $H_{1/3}$ are slightly big.
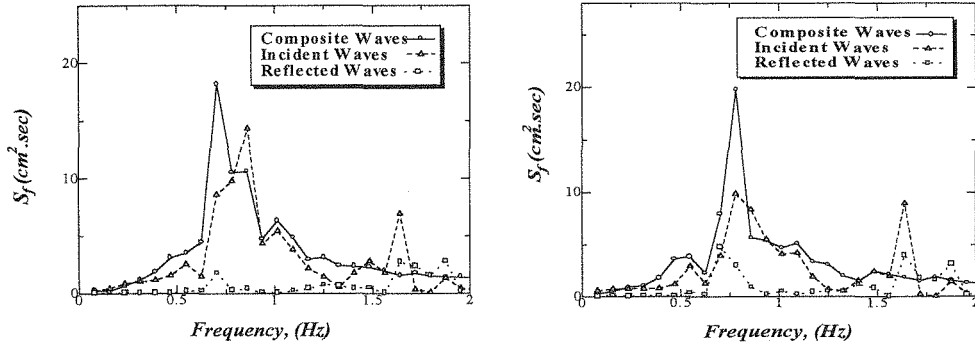
Fig. 9.   Spectrum analysis of wave spectra (Composite, Incident and Reflected) of incident and transmitted wave B-1.
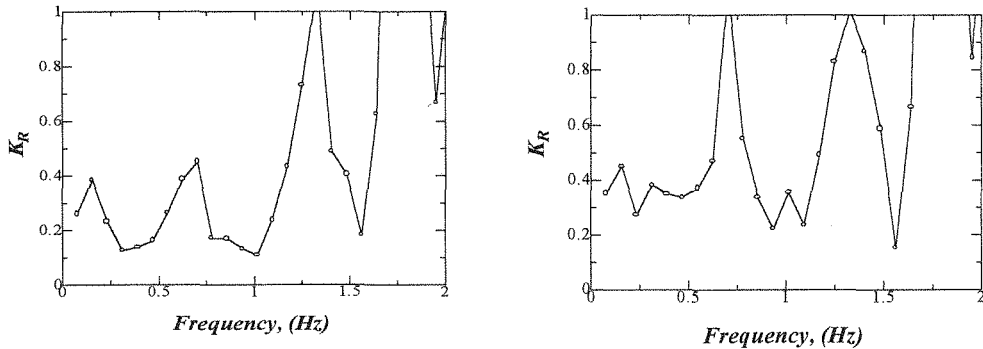
Fig. 10. Coefficient reflection of incident and transmitted waves.

Results of the incident wave height both in the seashore and in the transmission area were then employed by the Eq. (20) for obtaining the transmission coefficients. In the experiments were also arranged five distances between breakwater and dike. This idea was organized in order to know is variation of the distance influence to the values of wave transmission or not. By explanation of the Figs. 12 and 13, it may be taken the conclusion that the distance as mentioned has not effect to the transmission wave.
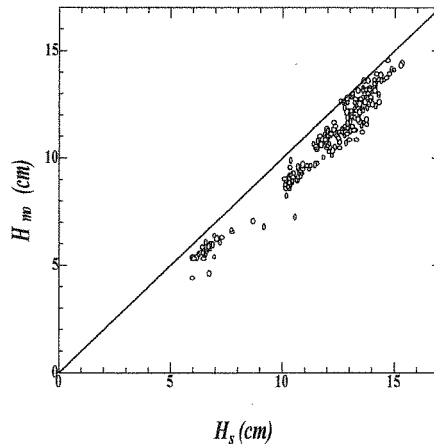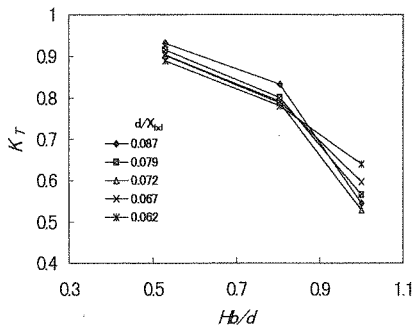
Fig. 11. The relationship between Hmo and $H_{1/3}$

Fig. 12. The relationship between breakwater's height-water depth's ratio and K_t.
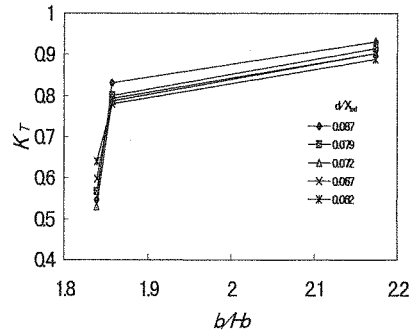

Fig. 13. The relationship between crest width-height ratio and K_t.

Nevertheless, the value of the transmission coefficents can be used as a reference in order to know the height of wave run up.

Table. 3. Average height of run up on breakwater crest height.

	$Ru_{1/3}(cm)$	$Ru_{1/10}(cm)$	$Ru_{max}(cm)$
Crest Height A	21.22	27.93	34.14
B	17.99	23.87	30.93
C	8.40	11.53	16.39

Table 3 shows the significant wave run-up, highest one-tenth of wave run-up and maximum wave run-up on the model dike from the water surface respectively. The R_{umax} of the highest crest height of breakwater (A type: submerged breakwater) does not even show that its overtopping pass really high over the crest of dike because of the occurred height of overtopping wave above the crest of dike was only about 7 cm. As the prototype height, it will be about 84 cm whereas, as shown in table 2, the significant run-up that

usually used in the design, the overtopping wave was not happened due its height lower than the crest of dike.

Therefor, it can be concluded that from the 3 types of breakwater crest height were applied in the experiment, the A type can be enough recommended as an alternative structure for increasing the effectiveness of Ariake reclamation dike.

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離岸堤に作用する反射波と透過波の性質に関する実験的研究
—有明海の海岸堤防をモデルとして—

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摘　　　　要

　透過性の離岸堤に作用する不規則波の反射波と透過波の性質について実験的に検討した．実験は，有明海の海岸堤防の1/12の模型で，Froude の相似側を用いて行った．離岸堤の高さを3種類変化させて実験をしたが，離岸堤の設置によって，透過した波も海岸堤防を超えることはほとんど防ぐことができた．

　波の反射率をスペクトル測定による方法と，Healy の方法によるものとで比較した結果，両者はよく一致した．

　この論文で得られた透過率は，有明海に離岸堤を設置する場合の設計指針を与えるに充分な結果を得た．