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BEARING CAPACITY AND DEFLECTION OF LATERALLY LOADED FLEXIBLE PILES

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Summary

The pile capacity, the bending moment variation with depth and the horizontal displacements at the load level of instrumented vertical single flexible model piles in loose sand, under lateral loads have been investigated. The results of these load tests are compared with theoretical estimates based on the concept of the effective embedment depth of equivalent rigid piles for ultimate and elastic cases. Reasonable agreement has been found between the observed and the predicted behaviour of flexible piles.

Key Words: bearing capacity, bending moment, deflection, flexible pile, instrumentation, lateral load, model test, sand

INTRODUCTION

Extensive studies on the behaviour of rigid piles ($K_r > 0.1$) under lateral loads have been reported (Brinch Hansen[1], Poulos and Davis[9], Meyerhof and Sastry[5, 6]). These studies have led to clear design concepts as far as rigid piles are concerned. However, in practice, most of the prototype piles are flexible ($K_r < 0.01$) which bend under the action of external forces. Attempts have been made recently to relate the behaviour of flexible piles in terms of equivalent rigid piles by introducing the concept of effective depth for both ultimate and elastic stages of loading (Meyerhof et al[8]., Sastry and Meyerhof[10, 11], Yalcin and Meyerhof[13]).

In continuation of the previous studies, the present investigation consists of instrumented model flexible piles buried in homogeneous loose sand and subjected to lateral loads. The bending moments in the pile shaft, the total load and the load level displacements under each load increment were recorded. The observations were analyzed to verify the applicability of the effective depth concepts to predict the pile capacity, maximum bending moment and horizontal deflections of flexible piles under lateral loads.

Table 1 Physical Properties of Piles

Depth D(mm)	D/B	Es(MPa)	Relative Stiffness of Pile K_r ($\times 10^{-4}$) Aluminium $Ep = 6.3\times10^4$MPa		Acrylic $Ep = 0.3\times10^4$MPa		Hard Rubber $Ep = 0.003\times10^4$MPa	
160	10	0.077	16,340.0	A1*	101.7	P1*	14	R1*
320	20	0.154	510.6	A2*	31.8	P2*	0.44	R2*
640	40	0.197	18.6	A3*	1.2	P3*		

Note: Ep=Modulus of elasticity of pile; B=16mm (Pile diameter);
Es=Weighted average horizontal secant modulus of soil in depth D;*=Pile number

MODEL TESTS

Soil Data

Dry Toyoura sand used in the tests was uniformly graded having effective size=0.12 mm and uniformity coefficient=1.67. The minimum and maximum void ratios of the sand were 0.61 and 0.96, respectively. The angle of internal friction ϕ determined from direct shear tests performed at a porosity of η=47 % was 31° (Koumoto and Kaku[3]). The horizontal secant modulus E_s of the sand, back calculated from rigid pile test results, was seen to vary linearly from zero at ground level to 365 kPa at a depth of 380 mm.

Pile Data

The model piles were made of aluminium, acrylic and hard rubber pipes having outside diameter B of about 16 mm and wall thickness of 1-4 mm. The piles were buried to a depth D of 160 mm, 320 mm and 640 mm in sand so that D/B values were 10, 20 and 40, respectively. The relative pile stiffness K_r ranged from 10^{-1}-10^{-5}. Each pile was instrumented with wire resistance strain gages staggered at a spacing of 40 mm on the outside skin of the pipes for the measurement of the bending moments. The gages were protected by enclosing the pile in polyolefin tubing. The details of piles tested are summarized in Table 1.

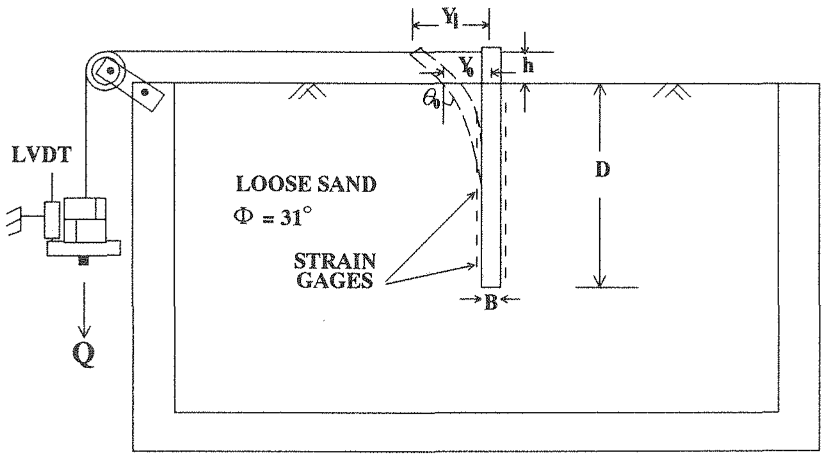

Fig. 1 Experimental Setup

Test Details

Sand was rained from a constant height of 500 mm in square steel box 480 mm×480 mm in section and 800 mm deep so that the placement unit weight was 13.72 kN/m^3. When the soil surface reached the required level, the pile was placed in position and the raining continued till the tank was full. The horizontal load was applied in 10–12 increments each being 1–10 N, depending on the estimated pile capacity, applied at a height of 25.4 mm above ground level as shown in Fig. 1. Under each load increment, the strain measurements at the gage locations were recorded by using Logger mate DL 1200, while the load level deflections were measured by a LVDT. Each load increment lasted till the rate of the horizontal settlement was practically zero and the gage outputs were constant.

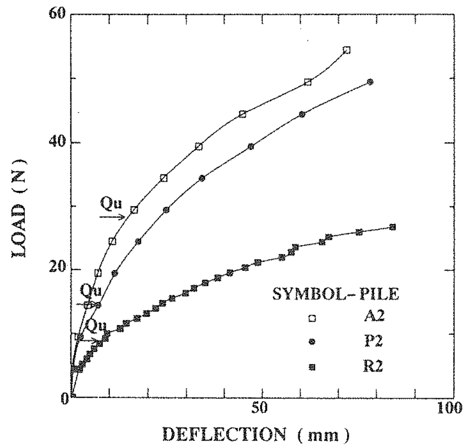

Fig. 2 Typical Load Deflection Curves.

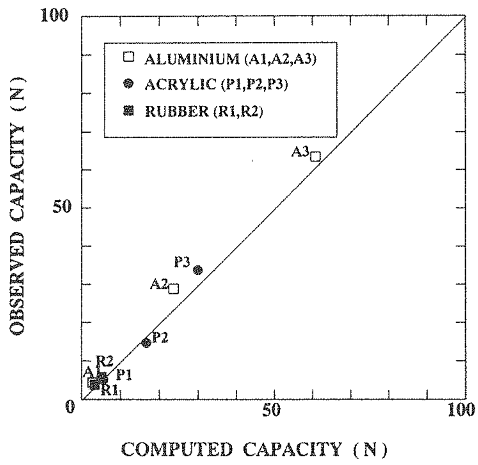

Fig. 3 Computed and Observed Capacities of the Piles.

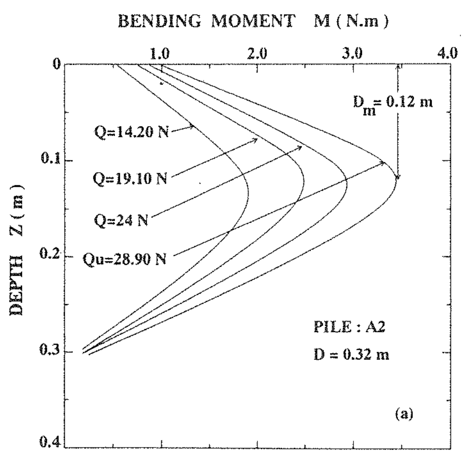

Fig. 4a Variation of Bending Moment With Depth and Load for Pile A2.

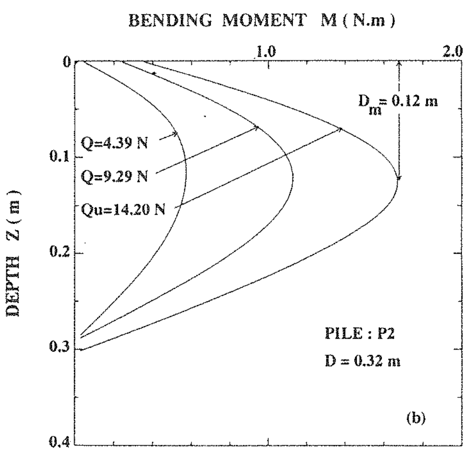

Fig. 4b Variation of Bending Moment With Depth and Load for Pile P2.

Test Results

Load deflection curves of the piles tested were found to be similar to those reported earlier from similar tests with large instrumented pile in homogeneous soils (Sastry and Meyerhof[10]) with some typical results presented in Fig. 2. The failure load was determined by extending the linear portion of the load deflection curve as suggested by Terzaghi[12] and the pile capacities are presented in Fig. 3. Typical results of measured bending moment variation with depth and load level for piles A_2 and P_2 are presented in Figs. 4 (a) and 4(b), respectively while the observed maximum bending moments in the piles are presented in Fig. 5. The measured depths of the points where the maximum bending moments were observed are presented in Fig. 6. Typical values of observed load level horizontal deflections for piles P_2 and A_3 are presented in Fig. 7(a) and 7(b), respectively.

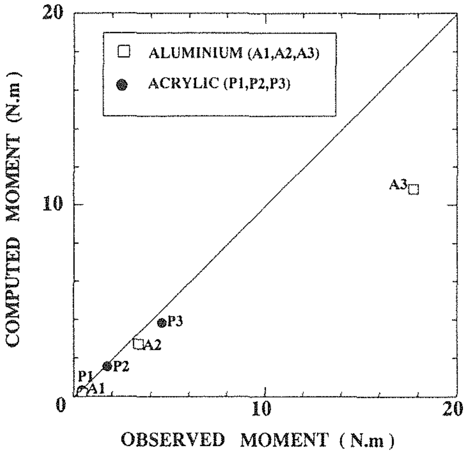

Fig. 5 Computed and Observed Maximum Bending Moments.

ANALYSIS OF RESULTS

Bearing Capacity

The effective depth concept applied to flexible piles under ultimate as well as elastic stages of loading is schematically represented in Fig. 8. The flexibility of the pile is measured by relative stiffness factor K_r defined by Poulos and Davis[9]

[1] $K_r = E_pI_p/E_sD^4$

where E_pI_p = flexural rigidity of the pile, E_s = weighted average horizontal secant modulus in embedment depth D. The pile behaves as a rigid element when $K_r > 0.1$ whereas it behaves as a flexible member when $K_r < 0.01$ with transitional behaviour between the above

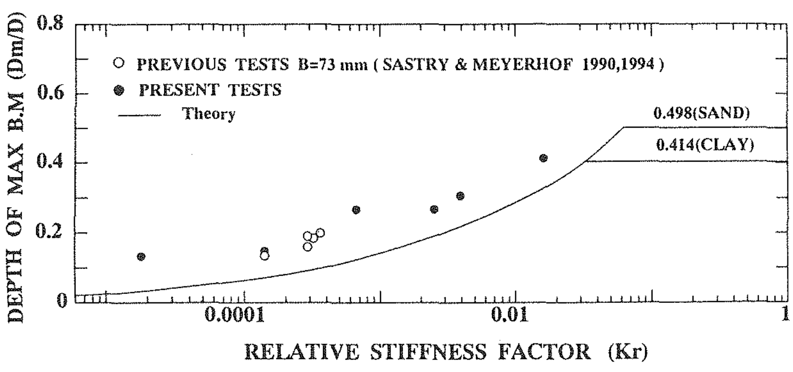

Fig. 6 Relation Between D_m/D and K_r.

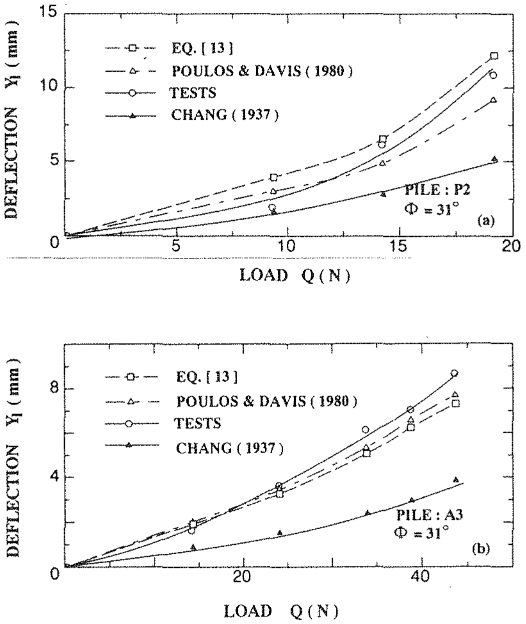

Fig. 7 Relation Between Load Level Horizontal Deflection Y_l and Load Q, for (a) Pile P2 (b) Pile A_3.

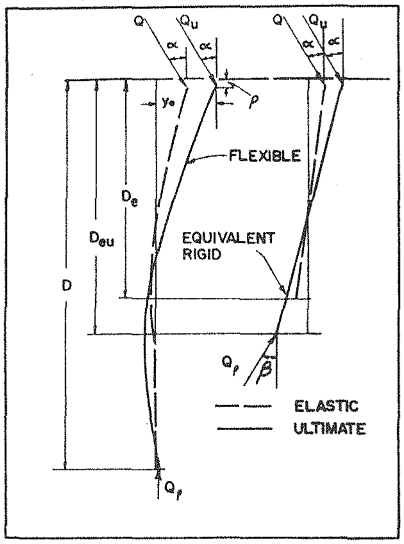

Fig. 8 Schematic Diagram of Flexible and Equivalent Rigid Piles.

two limiting values. A flexible pile of depth D can be considered as an equivalent rigid pile of ultimate effective depth D_{eu} for the computation of the pile capacity and the maximum bending moment whereas it can be treated as a rigid pile of elastic effective depth D_e for the estimation of deflections under working loads. The ratios D_{eu}/D and D_e/D are mainly controlled by the K_r value even though the variation of E_s with depth has some slight effect (Sastry and Meyerhof [11]). In the absence of structural failure of the pile, the ultimate lateral capacity of a flexible pile of embedment depth D in homogeneous sand is obtained by considering the equilibrium of an equivalent rigid pile of depth D_{eu} so that

[2] $Q_h = 0.125\,\gamma\,B\,D_{eu}^2\,K_b$

where γ = unit weight of the soil, K_b = earth pressure coefficient for the pile (Meyerhof et al.[7]) and D_{eu} is given by

[3] $D_{eu}/D = 1.65\,K_r^{0.12} \leqq 1$

Reasonably good agreement was seen between the computed and observed values of Q_h (Fig. 3).

Bending moments

In the design of piles, the magnitude and location of the maximum bending moment in the pile play a vital role. In the present case, the bending moment at ground level under failure load was given by $M_o = Q_h\,h$ where h is the eccentricity above the ground level where the load was applied. The bending moment was seen to increase parabolically up to a depth D_m where the maximum bending moment M_m was recorded, and was seen to decrease rapidly with further increase in depth (Figs. 4(a) and 4(b)). The non dimensional depth ratio

D_m/D was seen to be a function of K_r value and was decreasing as K_r was decreasing. The M_m value can be estimated once again by considering the equilibrium of an equivalent rigid pile of depth D_{eu} so that for h=0,

[4] $M_m = 0.36\, Q_h\, D_{eu}$

Reasonable agreement was seen between the computed and observed values of M_m (Fig. 5).

The elastic differential equation for the infinitely deep flexible pile ($K_r \leqq 0.01$) subjected to a lateral load at a height h above ground level is given by

[5] $E_p I_p (d^4 y_1/dx^4) = 0$ above the ground sufrace, and by

[6] $E_p I_p (d^4 y_2/dx^4) = p = -E_s . y$ below the ground surface (Chang[2]) where p is the net passive pressure on the pile, E_s is the weighted average secant modulus in depth D and y_1 and y_2 are the lateral deflections of the pile above and below the ground level at distance x. The above equations can be transformed as

[7] $d^4 y_1/dx^4 = 0$, and

$d^4 y_2/dx^4 + 4\beta^4 y_2 = 0$, in which $\beta = (E_s/4E_p I_p)^{1/4}$.

Solving these equations, the depth of maximum bending moment is obtained from (Koumoto et al.[4])

[9] $D_m/D = 1.414 K_r^{1/4} \tan^{-1}[1/\{1 + 1.414 (h/D) K_r^{-1/4}\}]$.

In the case of a rigid pile in clay, with h=0, $D_m/D = 0.414$ which is also obtained by considering the equilibrium of the pile. In the case of a rigid pile in sand the equilibrium considerations lead to $D_m/D = 0.498$. Theoretical values of D_m/D estimated from [9] agreed reasonably with the observed values (Fig. 6).

Displacements

The horizontal ground line displacement y_o and rotation θ_o of a flexible pile of depth D under a working lateral load $Q (Q = Q_h/3 - Q_h/2$ where Q_h is the pile capacity) acting h above the ground surface are estimated by replacing the flexible pile with an equivalent rigid pile of effective depth D_e given by (Sastry and Meyerhof[11])

[10] $D_e/D = 2.3\, K_r^{0.2} \leqq 1$

[11] $y_o = \{Q/(E_{se} D_e F_y)\}\{I_{yh} + I_{ym} . (h/D_e)\}$

[12] $\theta_o = \{Q/(E_{se} D_e^2 F_\theta)\}\{I_{\theta h} + I_{\theta m} . (h/D_e)\}$

where E_{se} = weighted average horizontal soil modulus in depth D_e, I_{yh}, $I_{ym} = I_{\theta h}$, and $I_{\theta m}$ are elastic influence factors for a rigid pile, F_y and F_θ are yield displacement and rotation factors, respectively for a flexible pile, (Poulos & Davis[9]). Considering average constant values of $I_{yh} = 7.5$, $I_{ym} = 9$ and $I_{\theta m} = 12$, y_o and θ_o were computed. The load level horizontal displacement y_l was approximately obtained from

[13] $y_l = y_o + \tan \theta_o . h$

This equivalent rigid pile method is referred to as method 1 and typical values of y_l are presented in Figs. 7 (a) and (b). y_o and θ_o values are also evaluated by considering [11] and [12], respectively for a flexible pile, in which D_e is replaced by D and the elastic influence factors appropriate to the K_r value are adopted (Poulos and Davis[9]). The y_l values are then obtained from [13] and are also presented in Figs. 7(a) and (b). This approach will be referred to as method 2.

An alternate method (method 3) was also used to calculate y_l values due to a working moment M and load Q acting at a height h above ground level on a flexible pile of depth D embedded in a soil with a uniform secant modulus E_s (Chang[2]),

[14] $y_l = [1/F_y][Q\{(1+\beta h)^3+0.5\}/(3E_pI_p.\ \beta^3)+M(1+\beta h)^2/(2E_pI_p.\ \beta^2)]$

with symbols as before. These y_l values are also shown for comparison in Figs. 7(a) and 7(b).

It was seen that the y_l values estimated from method 1 differ by about ±5% compared to those computed according to method 2 and both these methods provide a fair estimation of horizontal deflections. Method 3 was seen to underestimate the y_l values and provided a lower bound solution.

CONCLUSIONS

Analysis of measurements of the ultimate loads, bending moments along the pile shaft and load level displacements, of single instrumented flexible model piles, buried in loose sand, and subjected to horizontal loads, have led to a better understanding of the concept of effective depth for the design of flexible piles in homogeneous soils.

Methods have been suggested for obtaining the ultimate capacity of a flexible pile of depth D in sand and subjected to lateral load by replacing the flexible pile with an equivalent rigid pile of effective depth D_{eu}, the ratio D_{eu}/D being a function of the relative pile stiffness K_r.

The bending moment variation with depth can also be reasonably estimated from the concept of equivalent rigid pile subjected to theoretical soil pressure distribution on the pile shaft. The magnitude and position of maximum bending moment were reasonably estimated from the concept of equivalent rigid pile. The horizontal displacement of the pile head under working lateral load was reasonably esimated by replacing it with a rigid pile of elastic effective depth D_e, the ratio D_e/D being once again dependent on K_r, and using the expressions suggested earlier for a rigid pile.

Although, the present limited model pile tests on laterally loaded instrumented piles support the proposed concepts, it is felt that considerable further testing of model piles of wider range of relative stiffness of the pile, together with full scale instrumented pile tests under lateral loads is very much needed to verify the proposed concepts.

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横方向荷重下におけるたわみ性杭の支持力および変形

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摘　　　要

傾斜荷重を受けるたわみ性杭の設計に際しては，杭の横方向支持力および水平変位特性を知ることが重要である．たわみ性杭の鉛直支持力は，杭の相対剛性 K_r(＝E_pI_p/E_sD^4，ただし，E_pI_p：杭のたわみ剛性，D：杭の根入れ深さ，E_s：深さDにおける土の平均水平方向弾性係数)の大きさには無関係であるが，横方向支持力は K_r が減少するにつれて小さくなる．

一般に，$K_r \leqq 0.1$ の場合杭は剛性杭とみなされ，水平荷重下ではこの杭は杭中央部のある点を中心に回転する．これに対して，$K_r \leqq 0.01$ の場合杭はたわみ性杭とみなされ，水平荷重下ではこの杭は杭中央部で曲げ変形する．K_r が上述の両限界値の中間にある場合は，杭は水平荷重下において両杭の中間の挙動を示す．

これまで，杭の弾性変形状態および極限支持力状態における杭の有効根入れ深さの概念を導入して，たわみ性および剛性両杭の間の関係を求める試みを行っており，かなりの成果を得ている。

本論文は，ゆるい砂地盤に設置したたわみ性モデル杭の水平荷重下における支持力および変形特性を実験的に明らかにするとともに，提案した概念の適用性を検討したものである．